# Multiplicative Thinking

***"…the ability to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts"***

(Department for Education, Numeracy School Improvement)

| Micro Content |
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| **Cyclical pattern of 100-10-1 is repeated from ones to thousands** |
| **Cyclical pattern of 100-10-1 is repeated beyond 100s to millions** |
| **Ten times multiplicative relationship exists between places** |
| **The multiplicative relationship extends to numbers less than one, that is to the right of the decimal point** |
| **There is symmetry in the place value number system based around the ones place so that the patter in naming wholes is reflected in naming decimals** |
| **Double count by representing one group (e.g. hold up 4 fingers) and counting repetitions of that group, simultaneously keeping track of the number of groups and the number in each group** |
| **The multiplicative relationship between quantities is expressed as 'times as many' and 'how many times larger or smaller' a number is than another number** |
| **Numbers move a place each time they are multiplied or divided by 10** |
| **Basic number facts to 10x10 are recalled and patterns in number facts are investigated** |
| **Number facts can be extended by powers of 10** |
| **Multiplicative situations can be represented as equal-groups problems, comparison problems, combinations (Cartesian) problems and area/array problems** |
| **The multiplicative situation is understood (factor X factor = multiple) with the meanings of the terms clearly understood.** |
| **Multiplication arrays are used to visualise and represent multiplication situations** |
| **Division and multiplication are known as the inverse of one another** |
| **The communitive property of multiplication is understood and can be shown to be linked to arrays** |
| **Partition division involves finding the size of each group and quotition division involves finding the number of groups and can be also expressed in terms of factors and multiple** |
| **Quotition division can be considered in terms of fractions so that a quantity can be split by 'halving', 'thirding', 'fifthing' etc.** |
| **Prime and composite numbers can be linked to multiplicative arrays – prime numbers can be made only with a single row array** |
| **Distributive property of multiplication over addition is applied and shown by a multiplicative array** |
| **Multiplicative arrays are linked to the concepts or area and volume** |
| **Measurement units have the same multiplicative relationship as the Base 10 system** |

Micro Content from: Hurst, C., & Hurrell, D. (2014). *Developing the big ideas of number.*

**Collated by Ashley Gardner for Freeling Primary School 2019**

**Cartesian products can be represented symbolically and in tree diagrams**

***Common misconceptions:***

Students who have some knowledge of the multiplication facts to 100 and who perform simple multiplication and division procedures correctly are thought to be thinking multiplicatively.

Diagnostic testing reveals many rely on rote learning and/or a naive, 'groups of' understanding for multiplication based on repeated addition (often counting equal groups by ones), and as a consequence, tend to rely on memorised procedures for multiplying and dividing larger whole numbers and decimals.

***This could be due to/associated with:***

■ an inability to trust the count and see numbers as countable units in their own right, that is, view 6 items as 1 six ('a six') rather than 6 ones

■ poorly developed or non-existent mental strategies for addition and subtraction

■ an over-reliance on physical models to solve simple multiplication problems

■ a limited exposure to alternative models of multiplication.

To think multiplicatively students need to recognise the numbers 2 to 10 as countable units, count large collections efficiently, have developed mental objects for number and appreciate the advantages of representing multiplicative situations in terms of arrays and regions.

Arrays and regions support the shift from additive 'groups of' model to a factor-factor-product model which is needed to support fraction representation, the multiplication and division of larger whole numbers, fractions and decimals, and algebra. An awareness of the 'for each' idea or Cartesian product is also needed at this level to support work in statistics, data and probability.