Proportional Reasoning

"Students who have developed a broader range or ideas to support proportional reasoning use multiplicative strategies such as partitioning to solve problems…"
(Department for Education, Numeracy School Improvement)

Micro Content
Discuss situations involving relative comparisons (e.g. 7 fish in a fish-bowl, 7 fish in a garden pond, 7 fish in a lake) in terms of what remains the same & what is different
Compare and contrast the meaning of 'more' in different contexts, e.g. 'Harry had 17 football cards, Jamie had 23. How many more cards does Jamie have than Harry?' (additive/absolute thinking) compared to 'Harry traded 9 of his cards. Jamie traded 11 of his cards. Who traded more of their cards?' (Multiplicative/relational thinking)
Use Cuisenaire rods and/or pattern blocks to accurately describe relationships between rods or pieces (e.g. the triangle is 1 sixth of the hexagon and half of the rhombus)
Discuss the use of terms such as 'more' and 'times' and how many to identify problems requiring relational thinking (e.g. 'Which basketball game was closest, the Jets (49 goals) who defeated the Swallows (46 goals), or the Amazons (34 goals) who defeated the Warriors (31 goals?)'
Use a wide range of problem contexts/situations that require students to discriminate between those that require an absolute response and those that require a relational response
Support collaborative learning by modelling the types of arguments that need to be used to justify responses (e.g. use part-whole comparison which can be expressed as fractions, ratios or percentages)
Encourage students to devise their own relational problems using familiar contexts and sources such as newspapers, etc.
Develop a sense of 'per cent'
Build knowledge of fraction equivalence beyond halves and quarters to recognise that 1/8 is 12 ½% as it is half of 25%. 33 1/3 is 1 third, 10% is 1 tenth and so on. Model and practise the use of these in combination to find percentages mentally (e.g. for 45% of a quantity, find 50% and subtract half of 10%)
Explore the use of 'per cent' in an extended range of problems, discuss how percentage calculations can be performed on a calculator and why (e.g. to find 7 ½ % of $148, 530, multiply by 0.075 because 7 ½% is equivalent to 7.5 hundredths or 75 thousands)
Consider introducing ration more formally and the use of a wider range of complex proportional reasoning problems which require the use of 'percents'
Ensure that what is meant by statements such as, '3 times as big as' or 'half the size of' are understood, i.e. they can be modelled and interpreted relative to context
Introduce scale, use peg boards, dot paper, cm grid paper etc. – to enlarge and reduce shapes by simple scalar amounts. Discuss this in terms of what happens to corresponding sides (they are multiplied or divided by the same factor)
Provide opportunities to work with maps and scale diagrams, make thinking explicit, scaffold appropriate strategies for calculating or estimating distances
Practice map reading skills and strategies. Talk about the use of scales, construct scale drawings of the classroom, school grounds, students' homes and or/backyards. Discuss equivalent scales (e.g. 2cm to 150m is the same as 1cm to 75m)

Micro Content from: Hurst, C., & Hurrell, D. (2014). *Developing the big ideas of number.*
Collated by Ashley Gardner for Freeling Primary School

Explicitly link the use of scales to multiplication using the term 'scale factor', explore the impact of different scale factors, including scale factors less than 1
Understand relative proportion
Use a range of collaborative inquiry tasks so students discuss strategies for deciding 'what needs to be compared with what'
Provide opportunities for students to work on an extended range of problems involving different contexts and quantities expressed in different forms.
Interpret rational numbers
Review key fraction generalisations, in particular, the generalisation to support fraction renaming based on fraction diagrams
Consolidate rational numbers in all their forms by comparing, ordering, sequencing and renaming
Directly address the meaning of the symbols and forms of representations used.
Demonstrate the value of rational numbers in measurement (rates), chance and data (probability measures) and space topics (scale factors and map reading)

Students who have developed a broader range of ideas to support proportional reasoning use multiplicative strategies such as partitioning to solve problems involving simple proportion. These students can also work meaningfully with multiple representations of proportional relationships. For example, given a collection of 7 brown eggs and 5 white eggs, they recognise that the following representations can all be used to refer to different relationships in this context depending on what is viewed as the unit: 5:7, 7:5, 5:12, and 7:12.

Common misconception:

One of the reasons many Year 7 and 8 students experience difficulty interpreting and using ratios, rates and per cent, is that they have not yet acquired a capacity for proportional reasoning.