Generalising Tools

Common Misunderstanding

While considerably more is expected of students at this level in relation to their understanding and use of number than is expected at earlier levels most students are able to work with rational numbers to some extent and have an emerging appreciation of the real numbers. However, this is not necessarily the case when these numbers are represented by pro-numerals or used in expressions containing pro-numerals. For many, the very power and density of algebraic text can be the feature that renders it impenetrable.

There is an extensive body of research which has examined the various difficulties students experience with algebraic text, ranging from misunderstanding of the equal sign, and assigning literal meanings to letters (e.g. 3a interpreted as 3 apples) to viewing expressions as instructions to operate, rather than as objects that can be operated on in their own right (e.g. that 4 x -7 is an object that can be multiplied by any other number or pro-numeral).

While reading, interpreting, and working with algebraic text is one issue, constructing algebraic text to describe relationships is another area of difficulty for many students. A range of external representations (e.g. balances, concrete materials, graphs, diagrams, or tables of values) are typically used to explore patterns and relationships in school mathematics. Referred to as *intermediate sign systems* by Filloy and Sutherland (1996)[1], they variously serve to facilitate the construction of meaning for the conventional mathematical sign system, in this case, the "*algebra code*" (p.143). One of the difficulties here is that different conceptions arise from different representations and these may inhibit students' capacity to make connections between representations, generalise, or indeed, recognise when a previously learnt representation is inappropriate. For example, while it is meaningful to interpret 5 x □ = 20 as "*find the number which 5 must be multiplied by to equal 20"*, this interpretation (or intermediate sign system) cannot usefully replace x in the equation, 5x + 9 = 3x. Nor is it appropriate to expect that strategies that work for the former equation, such as "*back-tracking*", will work with equations like the latter where the unknown appears on both sides of the equation.

The difficulties experienced in making the transition from arithmetic to algebra may be due to/associated with:

- naïve understanding of the equal sign in terms of "*makes"* or the "*answer is*…";
- different interpretations of letters (Booth, 1988)[2] and/or a lack of knowledge about the conventions used to record generalised expressions (e.g. that multiplication is recorded as *3a* not *a3* or *3 x a*);
- limited understanding of the properties of numbers and operations (e.g. multiplication only understood in terms of *groups of*, division not seen as the inverse of multiplication);

[1] Filloy, E. & Sutherland, R. (1996). Designing Curricular for Teaching and Learning Algebra. In A. Bishop, McK. Clements, C. Keitel, J. Kilpatrick, & C Laborde. (Eds.) *International Handbook of Mathematics Education*, Part 1, pp. 139-160. Kluwer: Dordrecht

[2] Booth, L. (1988). Children's difficulties in beginning algebra. In A. Coxford & A. Shulte (Eds.) *The Ideas of Algebra K-12, 1988 Yearbook of the National Council of Mathematics*. NCTM: Reston, VA

- an inadequate understanding of arithmetic and/or an over-reliance on procedural solution strategies aimed at getting numerical answers;
- little/no experience in communicating mathematical relationships in words and/or translating relationships described in words into symbolic expressions, for example, "*s is 8 more than t*" or the "*Niger is three times as long as the Rhine*" (MacGregor, 1991, pp.95-97)[3] ; and
- limited access to multiplicative thinking and proportional reasoning more generally which restricts students' capacity to recognise and describe relationships in terms of factors.

Students are expected to be able to work meaningfully with a wider range of numbers and mathematical relationships in whatever form they appear, including equations, identities, inequalities, functions and relations.

A key indicator of the extent to which students are ready to engage with these curricula expectations is their capacity to deal with equivalent forms of expressions, recognise and describe number properties and patterns, and work with the complexities of algebraic text.

[3] MacGregor, M. (1992). *Making sense of algebra: Cognitive processes influencing comprehension*. Deakin University Press: Geelong.

6.1 Understanding Equivalence Tool[4]

MATERIALS

- True and False Cards (See Generalising Resources).
- T-shirts and Soft-drink Cards (see Generalising Resources).

INSTRUCTIONS

Place True and False Cards in front of the student.

Say: **"Some of these statements are true and some are false. Without calculating, can you tell me which ones are true and which are false and why?"** Note choices and whether or not student appears to be calculating. Explore reasoning where not obvious.

Place the cards in front of the student.

Say: **"Two T-shirts and two drinks costs $44. One T-shirt and 3 drinks cost $30. … How much does each item cost?"** If little or no response, and/or the student appears uncertain about how to proceed, ask:

"What is seems to be the problem here? What would make this easier?" Note response. If student produces a solution but his/her reasoning/working is unclear, ask:

"Can you explain how you worked that out please?" Note student's response.

[4] Adapted from Lange, J. de (1996) Using and Applying Mathematics in Education. In A. Bishop, et al (Eds.) *International Handbook of Mathematics Education*. Kluwer Academic Publishers: Dordrecht, The Netherlands (p.64)

6.2 Number Properties Tool

MATERIALS

- Sorting Cards (see Generalising Resources).
- A digital camera (if available) would be useful way to record card sorts efficiently.

INSTRUCTIONS

Place the cards in front of the student.

Say: **"Can you sort all of these cards into two groups please? Please tell me about each group?"** Note (or photograph) student's response, then ask:

"Is there another way you could sort the cards into two groups?" Note (or photograph) student's response and his or her reasons for the categorisation.

If student does not classify on the basis of primes/non-primes or factors/multiples, replace cards in front of the student.

Say: **"Can you find 4 or more cards that might be connected in some way?"** Note student's response and explore his or her reasoning.

Say: **"I'm thinking of a number … If I add 57 to it I will get 0. What number am I thinking about?"** Note student's response.

Say: **"I'm thinking of a number … If I multiply it by 8 I will get 3. What number am I thinking about?"** Note student's response and explore thinking as appropriate.

6.3 Pattern Recognition Tool[5]

MATERIALS

- 8 lengths of string (about 30 cm long).
- A ball of string (in case extra lengths are needed).
- A pair of scissors, pen.
- A copy of the String Cutting Record Sheet (see Generalising Resources).

INSTRUCTIONS

Place scissors and 5 lengths of string in front of the student.

Ask: **"Can you take one piece of string, fold it in half then cut across both strands once? How many pieces of string do you have?"** Ask student to record the result in the table on the String Cutting Record Sheet.

Continue with other pieces of string, making 2, 3, 4, and 5 cuts respectively and recording results in the table.

Ask: **"Can you see any patterns in the table? Could you describe them for me please?"** Note student's response.

"Without cutting any more string, can you use what you know about the patterns to complete the table? Can you tell me how you did that please?

If a general pattern not described in words or symbols, ask:

"Is there any way we could tell someone else how many pieces would be made by any number of cuts?" As you say this, indicate the "*n*" in the remaining top row cell of the table. Note student's response and explore his/her thinking.

If the general pattern was described fairly quickly in words or symbols, ask:

"What do you think might happen if instead of folding the string in half, it was folded into three parts and then cut like before?" Note student's response.

[5] This task was sourced from Cramer, K. (2001) Using models to build middle-grade students' understanding of functions. *Mathematics Teaching in the Middle School. 6*(5), 310-318

6.4 Understanding Algebraic Language Tool[6]

MATERIALS

- Set of Notation Cards (see Generalising Resources).
- Paper and pens for student use.

INSTRUCTIONS

Place Card 1 in front of the student.

Ask: **"Can you tell me what you think this means please? Could you think of a situation that this might describe? Can you write it another way?"** Note student's responses.

Place Card 2 in front of the student, point to 17 – 8.

Say: **"This is an expression for subtract 8 from 17. How would you express add 4 to 7*n*?"** Note student's response.

Say: "**And what does the "*n*" mean here? Does it mean anything, does it stand for anything, is it just a letter, or what?**" Note student's response and explore thinking as necessary.

Repeat with Card 3.

If answered confidently, place either Card 4 or Card 5 in front of the student, read the question slowly. Note student's response and explore as appropriate.

Place Card 6 in front of the student, read the question slowly, and indicate the three possible answer forms. Note student's response and explore reasoning as necessary.

Repeat with Card 7, stop if the student appears unwilling or unable to proceed.

[6] This task was sourced from Booth, L. (1988). Childrens' difficulties in beginning algebra. In A. Coxford & A Shulte (Eds.), *The Ideas of Algebra, K-12 – The 1988 Yearbook of the National Council of Teachers of Mathematics*, pp. 20-32, Reston, VA: NCTM

Generalising Advice

6.1 Understanding Equivalence Tool

A major source of student difficulties in algebra is the failure to recognise equivalence between different forms of the same relationship, or between the left and right sides of an equation. Early experiences in arithmetic, such as the representation of basic facts as equations (e.g. 6 + 8 = 14, 7 x 8 = 56, 24 ÷ 6 = 4 etc) and "*find the missing number*" tasks such as 3 + ? = 11, undoubtedly contribute to the view that the equal sign means "*makes*" or "*gives*" and that what follows is "*the answer*". That is, that symbolic expressions are seen as instructions to operate rather than as objects that can be operated on in their own right and expressed in a number of different ways. This view is widely regarded as one the major reasons why students experience difficulty in formalising and representing numerical or algebraic relationships in later years.

Students' responses to this task indicate the extent to which they can work with the notion of equivalence and understand some of the properties and conventions that underpin arithmetic expressions (e.g. recognising and using the commutative property and understanding why it does not apply to subtraction or division).

Observed Response	Interpretation/Suggested Teaching Response
Not all True/False Cards identified correctly, may need to calculate. Little/no response to the T-shirt problem, may try to represent problem in another form but unable to solve.	*May not appreciate the significance of order (commutativity) or how multiplication can be distributed over addition, and/or be able to recognise these when applied to fractions.* • Invite students to create equivalent expressions for well known facts, e.g. 3 x 4 = 2 x 6 = 24 ÷ 2 = 8 + 4 = 14 - 2 = 2 + 3 + 2 + 3 + 2 = and so on. Use *Fraction Walls*, *Number Line Diagrams* (see partitioning strategies in Partitioning Tools) and/or *Cuisenaire Rods* (where smallest rod is given a fractional name) to explore equivalent expressions involving fractions and decimals. • Explore consequences of reversing numbers, review order of operations (see Booker et al, 2003). • Encourage students to contrast and compare alternative models, e.g. 18 x 27 could be represented/thought of as 18 rows of 27 or 27 rows of 18, which is clearly larger than 19 rows of 18. • Explore simple word problems that lend themselves to algebraic reasoning, e.g. *Sally bought 2 snack bars and a drink for $6.15. If the drink cost $1.85, how much did she pay for one snack bar?* Encourage students to describe what they need to do in words and explore possible representations.

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6.1 Understanding Equivalence Tool

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Observed Response	Interpretation/Suggested Teaching Response
Identifies most of the True/False Cards correctly, may calculate some. Identifies the cost of a T-shirt and drink using a trial and error (guess and check) strategy.	*Suggests some understanding of underlying properties and conventions and an arithmetic approach to finding unknowns.* • Consider renaming problems such as 37 + 45 in an equivalent form to demonstrate how equivalent statements can be used to solve problems, e.g. 37 + 45 = 40 + 42 = 82 apply to problems like 4582 + 287 and generalise (e.g., p + q = (p + m) + (q – m). Explore other generalisations like this (e.g., see Irwin & Britt, 2005)[7] • Explore properties such as commutativity, distributivity, and associatively explicitly and examine other relationships such as whether or not (a + b) - c = (a - c) + b or (a x b) ÷ c = (a ÷ c) x b and why. • Use calculators to consolidate order of operations and the use of brackets to avoid ambiguities. • Discuss solution strategies for "*missing number*" problems that do not lend themselves to guess and check. Example: 11 + □ - 2 = 4 x □ • Explore the use of "*if ... then*" reasoning in problems like *Spiders and Beetles* (see Additional Resources).
Correctly classifies all True/False Cards (A, B and E true). Identifies the cost of each item relatively quickly and efficiently using some form of algebraic reasoning (e.g. recognises 1 T-shirt and 1 drink is half of $44 and uses this information to deduce that 2 drinks must be $8, so 1 drink is $4 etc, or solves using simultaneous equations methods).	*Suggests an understanding of equivalence as it is applied in non-conventional arithmetic settings and access to informal or formal strategies more closely related to algebraic thinking.* • Explore and justify concatenation, that is, the convention that the multiplication sign is omitted in situations like 4 x a to 4a. • Consider introducing/reviewing formal algebraic solutions to problems like the one above using linking words to show the reasoning involved. Example: If 11 + a – 2 = 4a then 9 + a = 4a so 9 = 3a (a subtracted from both sides) and a = 3 • Discuss the value and power of using a logical sequence of equivalent statements as a solution strategy.

[7] Irwin, K. & Britt, M. (2005). The algebraic nature of students' numerical manipulation in the New Zealand Numeracy Project. *Educational Studies in Mathematics, 58*(2), 169-188.

6.2 Number Properties Tool

An inadequate understanding of arithmetic, both in terms of the concepts that underpin the various representations of the four operations, and the properties that govern how we work with these more formally, is a related source of student difficulty in relation to algebra.

Student responses to this task indicate the extent to which they are able to recognise key differences between numbers (e.g. primes and non-primes) and how these might be described more formally (i.e. in terms of factors, multiples, and general expressions). They also reveal the extent to which students are inclined to think algebraically, that is, they are aware of and prepared to work with inverses and identities, as opposed to relying on arithmetic strategies such as 'guess and check' to solve equations.

Observed Response	Interpretation/Suggested Teaching Response
Predominantly sorts cards on the basis of fairly superficial features such as the number of digits, or magnitude (e.g. >100), may sort into odds and evens or identify factors/multiples as a common feature for 4 cards selected. Hesitant response to 'think of a number' questions, m.	*Suggests that multiplication (and division) not thought about in terms of factors, multiples, and divisors, relationship between fractions and division may not be fully understood.* • Explore problems involving different concepts for multiplication and division (see Multiplicative Thinking, Partitioning and Proportional Reasoning Tools). • Review the link between fractions and division i.e.: that $\frac{a}{b}$ means a ÷ b Using activities like *Fraction Sequences* (Stacey & MacGregor, 1997)[8] • Review the area and "*for each*" (or Cartesian Product) ideas for multiplication that underpin the notions of factors, multiples, divisors. • Engage in games and activities that focus on factors such as *Multo* (Maths 300, Curriculum Corporation, 2003) and *Multiples* (e.g. Stacey & MacGregor, 1997). • Review the definition of prime numbers and how "primeness" might be tested.

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[8] Stacey, K. & MacGregor, M. (1997). Building foundations for algebra. *Mathematics in the Middle School*, *2*(4), 253-260.

6.2 Number Properties Tool

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Observed Response	Interpretation/Suggested Teaching Response
Sorts cards on a more substantive basis (e.g. odds/evens or multiples of a particular number), provides at least three different sorts of this type, may sort on prime/composite basis. Correct response to first 'Think of a number' question (-57), may not be able to correctly respond to second question.	*Suggests a reasonable understanding of multiplication in terms of factors for whole numbers, may not appreciate role of identities and inverses in arithmetic.* • Use calculators to explore the impact of negative and fractional factors on quantities and expressions, apply in missing value problems, e.g. *Concentrates is mixed with water in the ratio 2 to 15 to make cordial. How much cordial could be made from 3 litres of concentrate*? • Explore the notion of inverses and identities for addition and subtraction using a wide range of numerical examples of the form: x ± ? = x –x + ? = 0 ? – x = 0 x + ? = 0 ? + x = 0 For whole numbers, integers, fractions and decimals. • Use a similar range of numerical examples to explore the notion of inverses and identities for multiplication and division. • Explore to role of factors in cancellation techniques, justify in terms of inverses and identities.
Provides more than 3 sorts based on multiples or factors, including prime/composite sort, able to describe bases for sorts using appropriate language. Answers last two questions correctly (i.e. 3 eighths or "*3 divided by 8*" or 0.375)	*Suggests generalised understanding of multiplication and division in terms of "factor.factor.product" and an understanding of inverses, may not be able to apply inverse operations to solve equations more formally.* • Relate the use of inverse operations to solving problems of the type, ax ± b = c ± dx where a, b, c, d progressively move from small whole numbers to larger whole numbers, integers, fractions and decimals. • Emphasise the value and power of recording which uses a logical sequence of equivalent statements to arrive at a solution (see 6.2 Advice above).

6.3 Pattern Recognition Tool

Working with number patterns has long been recognised as a means of introducing the notion of variable and more formal algebraic expression. However, there is often a disjunction between the physical models used to illustrate these patterns (e.g. matchsticks, tiles, wooded cubes, diagrams etc) and the formal language that is used to describe them in general terms. One of the reasons for this is that students will "*see*" and therefore "*count*" physical representations of patterns in different ways. More often than not, these ways are different to the more elegant and efficient ways "*seen*" by teachers of mathematics and textbooks.

In the advice below, reference is made to *Max's Matchsticks* (see Additional Resources), which encourages students and teachers to see that patterns can be viewed and described in different ways. Connecting pattern descriptions to how the elements might be counted is important as many students believe that finding "*the*" rule is a matter of guesswork or something that should just "*pop*" into one's head. In this task, the different strategies lead to different general descriptions (see below) which can be used to show equivalence and as a basis for a discussion about elegance and simplicity which justifies why one expression of the rule might be preferred over others.

Student responses to this task indicate the extent to which they are able to recognise, use, and describe a simple number pattern.

Observed Response	Interpretation/Suggested Teaching Response
Notices that the number of pieces of string "*goes up by 2*" with each cut, unable to say how many after 20 cuts without modelling and counting all or listing all elements in the sequence.	*Recognises difference between sequential terms but experiences difficulty describing functional relationships.* • Use idea of "*input/output machine*" and simple rules such as doubling, doubling and 1 more, take 3, squaring, and so on to establish the idea of a general rule and how it works. • Invite students to create a partial table of values for their own "*secret*" rule to play "*guess my rule*" with peers. • Use *Max's Matchsticks* (see Additional Resources) to acknowledge that people "*see*" patterns in different but ultimately equivalent ways. • As a follow-up to *Max's Matchsticks*, invite students to express the various strategies in words but referring to "*the number of squares*", e.g. for Jo's strategy, which counted the number of matches/square then subtracted the ones that had been double-counted, this might be "*4 times the number of squares minus 1 less than the number of squares*".

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6.3 Pattern Recognition Tool

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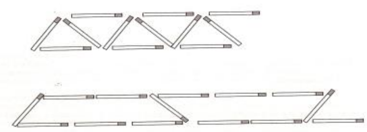
Observed Response	**Interpretation/Suggested Teaching Response**
Notices that the number of pieces of string "*goes up by 2*" with each cut, unable to say how many after 20 cuts without modelling and counting all or listing all elements in the sequence.	• Talk about what stays the same and what is different and develop general expressions for each strategy. Example: For Jo's strategy this would be: 4 x n – 9 or 4n – 9) Compare strategies and establish their equivalence. Discuss issue of elegance and simplicity and why 3n + 1 might be the preferred representation of this pattern.
Notices that the number of pieces of string increases by 2 with each cut, able to complete the table for 21 pieces of string (10) and 20 cuts (41) and describe pattern in words, may not be able to apply for 237 pieces of string or comment appropriately on what will happen when string folded in 3 parts.	*Able to describe simple patterns in natural language (e.g. double the number of cuts plus one), may not be able to describe more generally or use accurately.* • Use *Max's Matchsticks* activity (if not already used) to review the process of constructing general rules (i.e. from counting "*rules*" for particular cases to the general by identifying what stays the same and what changes and expressing this in words). • Use other matchstick patterns to explore different strategies and rule generation, example:  • Building block patterns can also be used to explore rule generation and application. • Investigate more general problems which can be explored diagrammatically. Example: Is there any way of determining the number of diagonals for an n-sided polygon? • Use rental situations where a certain amount is paid and then an hourly or kilometre rate is charged to show how generalised "*rules*" might be applied in the "*real-world*".

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6.3 Pattern Recognition Tool

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Observed Response	Interpretation/Suggested Teaching Response
Able to complete the table, states general rule in words and/or symbols, identifies what will happen when 3 pieces of string are cut (3n + 1) in words and/or symbols.	*Suggests a sound understanding of linear patterns and a capacity to use algebraic text in this context.* • Consider introducing pattern identification activities involving non-linear relationships, e.g. *Super Packer* (see Additional Resources), and growth/decay patterns (see Maths300). • Explore the value and power of algebraic text in different contexts, e.g. to problems such as: *The street numbers of 4 houses add to 3196.* *What are the numbers?* Discuss assumptions and problem representation. • Extend to problems involving non-linear relationships, e.g. *365 is an extraordinary number. It is the sum of 3 consecutive square numbers and also the sum of the next two square numbers. Find the numbers referred to.*

6.4 Understanding Algebraic Language Tool

Student responses to these tasks indicate the extent to which they are able to recognise and work with the conventions of algebraic text. In particular, the tasks explore how letters and departures from arithmetic forms of expression, such as the omission of the multiplication sign (concatenation or co-joining), are understood and used. For instance, many students believe that a + b is a more acceptable interpretation of *ab* than *a x b* (possibly on the basis of place-value).

While some of the tasks appear ambiguous (e.g. perimeter of incomplete polygon and spaceship journey), they were used with a large sample of Year 8 to 10 students (13-16 year-olds) in the UK (e.g. see Booth,1988)[9] to identify the extent to which students could derive and accept general expressions as "*answers*" to problems. This is an important, often under-recognised difficulty that significantly impacts students' capacity to work with general expressions. Where students view operation signs as instructions to "*do something*" and expect, as in arithmetic, to be able to perform those operation and arrive at a numerical answer, they will either make assumptions about what the numbers might be or just give up as they really do not see what the purpose or value is in using algebraic text.

This does not mean to say that students can't "*play the game*" of algebra, that is, look like they understand when they do not. For instance, many will correctly identify that 5a stands for "*5 times a number*" (Card 1), that 4 + 7n (or 7n + 4) represents "*add 4 to 7n*" (Card 2), and that 4m represents the sum of 4 m's (Card 3). However, some students will want to give numerical answers or approximations to the problems presented in Cards 4 and 5, and/or will not be able to formulate a correct expression for the Card 6 task (some version of p(a+ m), and/or identify the conditions under which the statements in Cards 7 and 8 are true (that is, p = y and n $\leq$ 2 respectively).

Observed Response	Interpretation/Suggested Teaching Response
May correctly respond to Cards 1, 2 and 3, but experiences difficulty with remaining Cards (e.g., may count sides of polygon shown and multiply by 2 to get numerical answer).	*Some understanding of the use of letters in simple additive contexts, may not understand equivalence, little/no understanding of the notion of variable.* • Check and consolidate as needed the convention of writing: a x b as ab Write such expressions in words, and practice translating expressions written in words into algebraic expressions. • Use tasks like the ones in Cards 4 and 5 to develop the notion of variable.

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[9] Booth, L. (1988). Children's difficulties in beginning algebra. In A. Coxford & A. Shulte (Eds.) *The Ideas of Algebra K-12, 1988 Yearbook of the National Council of Mathematics*. NCTM: Reston, VA

6.4 Understanding Algebraic Language Tool

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Observed Response	Interpretation/Suggested Teaching Response
May correctly respond to Cards 1, 2 and 3, but experiences difficulty with remaining Cards (e.g. may count sides of polygon shown and multiply by 2 to get numerical answer).	• Create and use variables in "*real-world*" contexts. Example: Investigate differences between different newspapers and magazines by identifying all the things that might be measured (e.g. column widths/lengths, words/page, area of different text forms, pictures, illustrations or headings /page, etc). Assigning a letter to each variable agreed upon, and then taking sample measures to compare and discuss. • Use activities like the ones described in 6.1 above to develop a deeper understanding of equivalence.
Correctly responds to Cards 1, 2, and 3, may respond appropriately to Cards 4 or 5 and 6, and/or partially recognise the conditions required for Card 7 (e.g. add **p** to left hand side and **y** to right hand side).	*Suggests an emerging understanding of how algebraic text is used, may be relying on arithmetic solution strategies rather than algebraic reasoning to solve problems.* • Review the use of renaming strategies to solve arithmetic problems and express as general relationships (see Irwin and Britt, 2005 and 6.1 above), e.g. xy = x(y + a) – xa from examples like 6 x 99, 8 x 789, or 25 x 9996. • Use "*consecutive number*" number problems. Example: *9 consecutive numbers sum to 936. What are the numbers?* "*Think of my rule*" activities and problems like the ones suggested in 6.3 above to consolidate the derivation of algebraic relationships.
Correctly responds to Cards 1, 2, 3, 4 or 5, and 6, may not be able to explain reasoning, recognises the conditions required for Card 7, but unable to do so for Card 8.	*Suggests reasonable understanding of algebraic conventions may not understand inequalities and/or be able to manipulate/transform equations or inequalities.* • Check and consolidate as needed, students' understanding of inverse relationships (see 6.2) and how these might be applied to solve simple linear equations.

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6.4 Understanding Algebraic Language Tool

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Observed Response	**Interpretation/Suggested Teaching Response**
Correctly responds to Cards 1, 2, 3, 4 or 5, and 6, may not be able to explain reasoning, recognises the conditions required for Card 7, but unable to do so for Card 8.	• Relate the use of inverse operations to solving problems of the form ax ± b = c ± d where a, b, c, and d progressively move from small whole numbers to larger whole numbers, integers, fractions and decimals. • Introduce/consolidate inequalities working from words to symbols using examples like the one shown in Card 8.
Correctly responds to all Cards.	*Suggests sound understanding of algebraic notation and conventions.* • Extend pattern recognition and identification to non-linear relationships and graphical representations. • Use problems like the following to consolidate the understanding that "*answers*" can be expressed in general terms: *A bacterium can reproduce itself every 20 minutes. If a colony of bacteria was put into a breeding dish at 10:05 am. How many bacteria would there be by 3:45 pm?* • Explore a wider range of situations and problems involving inequalities. • Explore the range of strategies that might be used to solve problems such as *Spiders and Beetles* (See Additional Resources), e.g. guess and check, draw a diagram, make a table, or use "*if … then*" reasoning. Use this and similar problems, to demonstrate the value of algebraic representation and reasoning (e.g. can solve simultaneously and/or graphically).