Partitioning Tools

Common Misunderstanding

There is little doubt that a considerable proportion of Year 5 and 6 students experience difficulty with fractions, decimals and percent. A major factor contributing to this is that many students misinterpret the meaning of the denominator. Also, while students may exhibit an intuitive understanding of proportionality in terms of the *out of* idea, this is limited to familiar contexts and proper fractions (e.g. 3 quarters of a pizza or the fraction of red smarties in a packet of smarties). Few students at this level see fractions as numbers which can be arrived at by partitive division (e.g. 3 pizzas shared among 4) and 'live' uniquely on the number line as measures.

This could be due to/associated with:

- viewing the denominator in the same way as the numerator (i.e. as a count or 'how many' number, rather than an indication of "*how much*");
- a limited exposure to practical experiences that show what happens as the number of parts are increased and how fractional parts are named;
- a *groups of* only idea for multiplication and division; and;
- little or no access to strategies that support the construction of appropriate fraction representations.

Students need to be able to work meaningfully with a wider range of numbers. In particular, they need to have established a meaningful basis for thinking about rational numbers in whatever form they appear (e.g. proper fractions, mixed fractions, decimal fractions, and percentages). This requires the recognition that equal parts are required; that the number of parts is related to the name of the part (i.e. fifths for 5 parts, sixteenths for 16 parts); that as the number of parts increases, each part becomes smaller; and that fraction representations are created by partitioning discrete or continuous quantities into equal parts (see *Partitioning* paper in Additional Resources).

Understanding the relationship between fractions and partitive division is essential for fraction renaming (equivalent fractions). In particular, students need to recognise how the *region* idea for multiplication is related to fraction diagrams, for example, thirds (3 parts) by quarters (4 parts) produces twelfths (12 parts), and how increasing/decreasing the number of parts can be understood in terms of *factors*, for example, recognising that 3 parts (thirds) increased by a factor of 4 (as a result of halving and halving again) produces 12 parts (twelfths).

Key indicators of the extent to which students have developed an understanding of fractions and decimals is the extent to which they can construct their own fraction models and diagrams, and name, record, compare, order, sequence, and rename, common and decimal fractions.

4. Partitioning

4.1 Equal Parts Tool

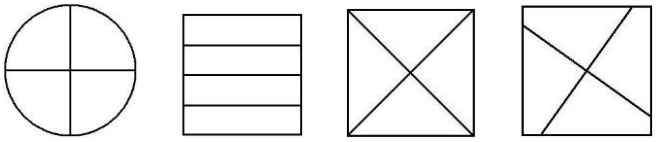
MATERIALS

- 8 Equal Parts cards (see Partitioning Resources).
- Copy of Worksheet 1 (see Partitioning Resources) cut in half along the dotted line.
- Pencil and eraser (if needed).

INSTRUCTIONS

Place the Equal Parts cards in front of student.

Say: **"All of these squares and circles have been divided into 4 parts. Which ones are fractions?** If not done spontaneously, suggest student places the cards into two groups. Note student's response and ask them to explain their reasoning by asking why cards were placed in each pile, e.g. fraction cards are:



If at least 2 cards correctly identified as showing fractions, place the top half of the worksheet in front of the student.

Say: **"Can you shade this rectangle to show 2 fifths please? Can you tell me why you did it that way?"** Note student's strategy. If student appears hesitant, ask…

"What seems to be the problem here?" Note student's response.

Place the bottom half of worksheet in front of the student.

Say: **"Can you shade this rectangle to show 2 fifths please? Can you tell me why you did it that way?"** Note student's strategy. If student appears hesitant, ask…

"What seems to be the problem here?" Note student's response.

4.2 Fraction Naming Tool

MATERIALS

- A square piece of paper (approximately 20 cm x 20 cm) folded into 6 equal parts.
- Large sheet of Butcher's paper.
- 24 coloured counters.

INSTRUCTIONS

Place folded paper in front of the student.

Say: **"Can you unfold that please and tell me what you notice?"** Note student's response. If a student does not comment on the number of parts or equality of parts…

Say: **"Do you know what we call these equal parts?"** If no response or a non-fraction name given…

Say: **"Another student I talked to said these were eighths, do you agree? Why/why not?"**

Place the sheet of Butcher's paper in front of the student and ask him/her to fold it in half and in half again.

Say: **"Without unfolding the sheet of paper, how many parts has the sheet of paper been folded into? Do you know what these parts are called?"** Note student's response.

Ask: **"If you folded it in half again, can you predict how many parts there would be? What would they be called? How do you know that?"** Note student's prediction then ask him/her to fold the paper in half again. If student predicted 6 parts or sixths, unfold, check then refold. Ask the student to continue halving. Stop student after he/she has folded three more times (64 parts) and ask:

"Without unfolding the sheet of paper, about how many parts do you think there might be now? If there were that many parts, what would they be called?" Note response and invite the student to unfold the paper. When completed, ask:

"How many parts did you make?" Note student's strategy, in particular, whether they count by ones or use the number of rows/columns, and then ask:

"What would these parts be called?" If correct (sixty-fourths), ask:

"Which is bigger, three of these parts (indicating the sixty-fourths) **or 3 eighths? Why?"** Note response.

Continued on next page

4.2 Fraction Naming Tool

Continued from previous page…

Place coloured counters in front of student.

Say: **"There are 24 counters here"** … take 4.

Say: **"This is my share"** … give student 6.

Say: **"This is your share, what fraction of the counters did I get? What fraction of the counters did you get? Do we have equal shares?"**

4.3 Fraction Making Tool

MATERIALS

- 24 Counters.
- A ball of plasticine (enough to make a 'pizza' about 6 cm in diameter and 0.5 cm thick).
- An icy-pole stick or plastic knife (not a ruler as this might suggest measuring which could mask partitioning behaviour).
- 4 small kindergarten squares of the same colour.
- A copy of the Fraction Making Worksheet (see Partitioning Resources), cut in half along the dotted line.
- Rectangular "*tile*" approximately 3 cm by 5 cm made of thick card or wood (this needs to be thick enough not to be folded easily).
- Paper, pencil and eraser.

INSTRUCTIONS

Place the 24 counters in front of the student.

Say: **"Could you give half of those to me please?"** Note student's strategy. Regroup counters to a single group.

Say: **"Now, could you give me 1 third please?"** Note student's strategy. If done fairly easily, regroup counters and ask:

"What if you had to give me 5 eighths, how would you do that?"

Place the ball of plasticine in front of the student.

Say: **"Can you make that look like a pizza please?"** Once this as been done, hand the student an icy-pole stick and ask:

"Can you show me where you would cut that to make 8 equal pieces?" Note student's cutting strategy.

If the "*pizza*" has been "*cut*" reasonably accurately into 8 equal pieces…

Say: **"If 3 quarters of this pizza was eaten, can you show me how much was eaten?"** Note student's response.

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If the student successfully completed the "*pizza*" task, roll plasticine back into a ball and place in front of student.

Say: **"Could you use this to show 5 thirds please?"** Note student's response, in particular, what is regarded as the unit.

If student appears hesitant, remove plasticine, place kindergarten squares in front of student, and ask:

"Could you use this paper to make 2 and 5 sixths please?" Note student's response.

If either one of these tasks is completed reasonably well, place the top half of the worksheet in front of the student.

Say: **"Can you divide this rectangle into 3 equal parts please? Can you tell me why you did it that way?"** Note student's strategy, in particular, whether or not halving is used to estimate. If parts reasonably equal.

Say: **"Do you know what these parts are called? Why?"** Note student's response.

Place the bottom half of the worksheet in front of the student.

Say: **"Can you divide this line into 5 equal parts please? Can you tell me why you did it that way?"** Note student's strategy (in particular, if they appeared to count from left to right or estimated using a halving strategy). If parts reasonably equal, ask:

"Do you know what these parts are called? Why?" Note student's response.

If rectangle and line partitioned more or less accurately, place paper, pencil and rectangular "*tile*" in front of the student.

Say: **"If this tile is 2 thirds of a whole, could you draw the whole please?"** Note student's strategy, in particular, whether or not halving is used to estimate.

Draw a line (approximately 12 cm long) on the student's paper, label one end 0 and the other end 6 fifths (written symbolically with a horizontal bar).

Say: **"Please mark the line to show where 1 would be. Can you tell me why you did it that way?"** Note student's strategy (in particular, if they appeared to count from left to right or estimated using a halving strategy).

4.4 Fraction Recording Tool

MATERIALS

- Birthday Cake (see Partitioning Resources) laminated and cut into 6 pieces.
- Fraction Recording Cards (see Partitioning Resources) laminated and cut into individual shapes (represents 2 and 3 eighths).
- 15 square tiles, 6 of one colour (e.g. blue) and 9 of another colour (e.g. yellow).
- Pen and paper.

INSTRUCTIONS

Group the pieces of the "*Birthday Cake*" into a "*cake*" in front of the student, remove 1 piece.

Say: **"If this piece of cake was eaten, what fraction of the cake would be left?"** Note student's response. If correct, ask:

"How would you write that as a fraction?" If written symbolically, circle the numerator and as you are doing this…

Say: **"What has this got to do with what you have got there**? Note student's response then circle the denominator and **repeat the question**, noting the student's response.

Re-assemble the "*cake*" (if necessary).

Say: **"If 2 thirds of the cake was eaten, can you show me how much was eaten?"** Note student's response. If correct, ask:

"How would you write that as a fraction?" If written symbolically, circle the numerator.

Say: **"What has this got to do with what you have there?"** Note response, then circle the denominator and **repeat the question**, noting the student's response. If student appears hesitant, record 2/3 as a fraction (vertically aligned with horizontal bar).

Say: **"Some people might record it like this, would they be right? Why?"** If student agrees, circle each of the numbers in turn and repeat the earlier questions, otherwise proceed to the next question.

Continued on next page

4.4 Fraction Recording Tool

Continued from previous page

Place the Fraction Recording Cards in front of student, point to one of the squares.

Say: **"This is 1 whole. Can you write what is shown here as a fraction please?"** Note student's response. If written symbolically, circle each of the numbers recorded in turn and as you do this ask:

"What has this got to do with what you have got there? Note student's response to each number circled.

Place the 15 square tiles in front of the student.

Say: **"Please write down what fraction of the tiles are blue."** If 6/15 recorded, circle the 6.

Say: **"What has this got to do with what you have there?"** Note response, then circle the 15 and repeat the question, noting the student's response. Ask:

"Is there another way we could write that as a fraction?" If 2/5 recorded, circle each of the numbers in turn and repeat the earlier questions. If student appears hesitant, record 2/5 as a fraction (vertically aligned with horizontal bar).

Say: **"Some people might record the fraction of blue tiles like this, would they be right? Why?** If student agrees, circle each of the numbers in turn and repeat the earlier questions.

4.5 Decimal Fraction Naming and Recording Tool

MATERIALS

- Standard tape measure (showing cm and mm).
- Length of non-stretch paper streamer exactly 1.45 metres long.
- Decimal Naming and Recording Cards (see Partitioning Resources).
- Paper and pen.

INSTRUCTIONS

Place the length of streamer and the measuring tape in front of the student and ask:

> **"If I hold the streamer and the measuring tape at this end** (make a point of lining up the end of the streamer and the zero on the tape measure)**, could you measure the length of the streamer as accurately as possible please? How would you write that?"** Note student's response. If 145 cm recorded, ask:
>
> **"Could you write that in metres please?"** If 1.45 m recorded, circle the 1.

Say: **"What has this got to do with what you have there? Can you show me on the tape measure?"** Note response, then circle the 4 and repeat the questions noting the student's response. If reasonable response provided, circle the 5 and repeat the questions noting the student's response. If student appears hesitant…

Say: **"Another student I talked to wrote this** (record 1.45 on paper) **what do you think that tells us about the length of the string? Why? Can you show me on the tape measure?"** Note student's response.

Place the Decimal Card showing 6 tenths in front of the student.

Say: **"Can you tell me what fraction of this rectangle is shaded? Could you write that down please?"** If written as a common fraction, ask:

> **"Is there another way we could write that?"** If decimal form not offered, record 0.6 on student's paper and ask:
>
> **"Would this be correct? Why do you think that?"** Note student's response.

If answered correctly, repeat with the card showing 63 hundredths. In this case, if 63% not offered at some point, record on student's paper and ask:

> **"Would this be correct? Why do you think that?"** Note student's response.

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4.5 Decimal Fraction Naming and Recording Tool

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Place the Decimal Card showing 1 and 79 hundredths (1.79) in front of the student.

Say: **"If this is 1 whole** (point to fully shaded rectangle)**, can you tell me what number is shown by the shaded areas altogether? Could you write that down please?"** If written as a whole number and a common fraction, ask:

"Is there another way we could write that?" If not offered, record 1.79 on student's paper and ask:

"Would this be correct? Why do you think that?" Note student's response. If correct, circle the 7 and ask:

"What has this got to do with what is shown there?" Indicate 1.79 fraction card. Note student's response.

Place the Number Line Card in front of the student.

Say: **"At what point does the green line end?"** Note student's response. If answered correctly, **ask the same question for the purple line** noting student's response. If incorrect or very general (e.g. "*it's five and a bit*"), ask student to explain his/her thinking.

4.6 Comparing and Ordering Tool

MATERIALS

- Fraction Comparison cards (see Partitioning Resources).
- Fraction Mat and Fraction Mat Cards (see Partitioning Resources).
- Decimal Word cards (see Partitioning Resources).
- Decimal Symbol cards (see Partitioning Resources).

INSTRUCTIONS

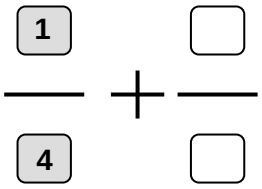
Say: **"I'm going to ask you a question which I would like you to think about in your head … Ready? Which is bigger, three quarters or four ninths? How do you know?"** Note student's response.

Place the Fraction Comparison cards in front of the student.

Say: **"Which one of these is smaller? How do you know?"** Note student's strategy.

Place the Fraction Mat and the Fraction Mat Cards in front of the student. Use the number cards to make 1 quarter on the mat as shown.

Say: **"These cards can be used to make fractions like this** (indicate the 1 quarter)**. Please make two fractions that when added together are as close as possible to one."** Note student's choice.



Place the Decimal Word cards in front of student.

Say: **"Please put these in order from smallest to largest."** Indicate that the smallest card should go to the student's left. Once student is satisfied with order, ask him/her to explain their reasoning.

Place Decimal Symbol cards in front of the student.

Say: **"Can you put these in order from smallest to largest please?"** Indicate that the smallest card should go to the student's left. Once student is satisfied with order, ask him/her to explain their reasoning.

4.7 Rational Number Sequencing Tool

MATERIALS

- Length of rope (approximately 0.5 cm in diameter and 1.5 metre long).
- 10 clothes pegs.
- Rational Number Cards (see Partitioning Resources).
- Negative Number Cards (see Partitioning Resources).

INSTRUCTIONS

Place the Rational Number Cards and rope in front of the student. Peg the "**0**" card at one end of the rope and the "**2**" card at the other end.

Say: **"If this end of the rope is 0 and this end is 2, can you peg the cards on to the rope to show where the numbers live? Can you tell me why you did it that way?"** Note student's response, in particular, whether or not the fraction is interpreted as an operator or as a number (e.g. is ¾ interpreted as ¾ of the rope of or as a number in its own right less than 1, similarly with 50%).

Using the Negative Number cards, place the string in front of the student. Peg the "**-3**"card at one end of the string and the "**3**" card at the other.

Say: **"This end of the rope is negative (or minus) 3 and this end is 3."** Give the "**0.5**" card to the student.

Say: **"Can you peg that on the rope to show me where this lives? Can you tell me why you did it that way?"** Repeat with the following cards in order, -2.5, 2/3 and -1¾. Note student's response.

Partitioning Advice

4.1 Equal Parts Tool

Although it is commonly assumed that Year 2 to Year 4 students "*see*" what most adults "*see*" in fraction diagrams, this is not generally the case. Indeed, many students do not look at the relevant attribute of area, but at the lines, vertices and shapes represented. This is not surprising given that for the past 3 to 6 years, they have been asked to focus on these attributes to recognise the properties of two dimensional shapes. Recent research suggests that even though students may correctly solve fraction problems (e.g. If 3 pizzas are shared among 4 people, what fraction of pizza will each person receive?), they are unable to recognise or draw a correct representation of this (SNMY, 2005).

This task is designed to evaluate the extent to which students' recognise the importance of equal parts in fraction representations.

Observed Response	Interpretation/Suggested Teaching Response
Little/no response.	*May not understand task.* • Use plasticine or play-dough to make recognised wholes (e.g. a "*pizza*", rectangular "*cake"* or "*sausage*") cut as appropriate to distinguish between equal parts and unequal parts, emphasising that those with equal parts show fractions. Use models to make and name fractions. • Extend to paper-folding, creating equal and unequal parts to distinguish between fraction and non-fraction models, and link paper models to diagrams.
Identifies at least some fraction cards correctly but includes the following card.  May attempt to shade 2 fifths but likely to do this without reference to the number (6) or nature (unequal) of the parts.	*Suggests equal parts recognised to some extent but can be over-ridden by strong, misleading perceptual information.* • Investigate "*fair shares*" using a variety of physical models (e.g. plasticine, paper, streamers, "*Smarties"*), discuss relevant attribute (e.g. area or length) and explore ways in which equality of parts can be tested (e.g. cut up and laid on top of one another). • Consider introducing *halving* and *thirding* partitioning strategies via paper-folding (see *Partitioning* paper in Additional Resources) with a view to scaffolding students' capacity to generate their own fraction models and diagrams.

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4.1 Equal Parts Tool

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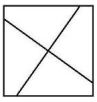
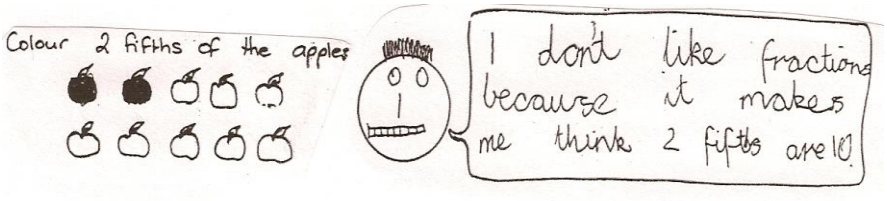
Observed Response	Interpretation/Suggested Teaching Response
Identifies 3 fraction cards but not the following:  Some attempt to shade 2 fifths, recognises at least one issue making this difficult (i.e. the number and/or nature of parts), e.g. may eliminate one part from the first diagram or guess to estimate 2 fifths.	*Suggests equal parts recognised to some extent and some awareness of the importance of the number of parts.* • Investigate "*fair share*" as above, extend to non-conventional representations such as the one shown here and the means by which these can be tested (see above). • Introduce/consolidate *halving* and *thirding* partitioning strategies (see above). • Consider introducing *fifthing* strategy (see *Partitioning* paper in Additional Resources) using a variety of materials.
Identifies all fraction cards and recognises that both the number of parts (first diagram) and the equality of the parts (second diagram) are problematic, may attempt to deal with these in some way.	*Recognises the importance of equal parts and the total number of parts irrespective of representation.* • Extend the use of the *halving*, *thirding* and *fifthing* partitioning strategies (see *Partitioning* paper in Additional Resources) to expand students' capacity to generate their own fraction models and diagrams irrespective of representation.
Identifies all fraction cards correctly and shades to show 2 fifths fairly accurately (i.e., slightly more than 2 sixths for the first diagram and to the mid-point of the third part for the second diagram).	*Suggests access to meaningful partitioning strategies and an understanding of how fractions are named and represented.* • If not already recognised, make partitioning strategies explicit and use to make and name a wider range of fractions using region diagrams and open number lines (e.g. tenths, twelfths, fifteenths etc). • Use partitioning strategies to re-name common fractions, developing the generalisation: *If the total number of parts is increased by a certain factor, then the parts shaded or required are increased by the same factor*. • Consider introducing/consolidating decimal fractions to thousandths by successively "*tenthing*", use to compare, order, sequence and rename decimal fractions.

4.2 Fraction Naming Tool

Many students do not make the connection between the number of parts or shares and the name of the parts. This is not surprising given that the most commonly recognised fractions (halves and quarters) do not fit the pattern which uses ordinal number names to indicate parts (e.g. three parts, *thirds*). The use of ordinal names can also be confusing as is indicated by the following example, where the "*fifth*" are interpreted as being equivalent to "*five*", presumably on the basis that identifying the fifth in line is associated with counting to five.



Students may also fail to recognise the important generalisation that *the larger the number of parts, the smaller they are in relation to the whole.* This misconception can arise as a consequence of an undue emphasis on discrete models (e.g. 3 out of 20 smarties) where the size of the actual object (a single smarty in this case) does not change if the size of the collection is changed but it's relationship to the whole does (e.g. 3 out of 50 smarties). That is, discrete models can lead to a confusion between wholes and parts which may mask the relationship between them. This is not to say they should not be used, but that they should be used in conjunction with continuous models where this generalisation is more readily observed.

This task is designed to evaluate the extent to which students' recognise how fractions are named and appreciate the impact of increasing the number of parts on the relative size of the part.

Observed Response	Interpretation/Suggested Teaching Response
Little/no response, may correctly respond to first task after prompt (e.g. says not eighths as 6 parts or sixths), incorrect or irrelevant responses to some or all aspects of the paper folding and counters tasks, e.g. uses remaining counters (14) as the denominator in naming shares.	*May not understand task* • Use paper-folding to make and name parts in the halving family (e.g. halves, quarters, eighths, sixteenths, thirty-seconds, sixty-fourths etc)

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4.2 Fraction Naming Tool

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<table>
<tr><th>Observed Response</th><th>Interpretation/Suggested Teaching Response</th></tr>
<tr><td>Little/no response, may correctly respond to first task after prompt (e.g. says not eighths as 6 parts or sixths), incorrect or irrelevant responses to some or all aspects of the paper folding and counters tasks, e.g. uses remaining counters (14) as the denominator in naming shares.</td><td>• Make a table that records the number of parts and the name of the parts, e.g.
<table><tr><th>Number of Parts</th><th>Name</th></tr><tr><td>1</td><td>whole</td></tr><tr><td>2</td><td>halves</td></tr><tr><td>4</td><td>quarters</td></tr><tr><td>8</td><td>eighths</td></tr></table>• Discuss use of ordinal names, inconsistencies and peculiarities (e.g. thirty-seconds sounds like 30 seconds as in a measure of time) and the generalisation about the number of parts in relation to their size</td></tr>
<tr><td>Names parts as sixths for first task may predict sixths and/or use an additive strategy to determine the number of parts in paper-folding task (64). May not recognise 3 eighths as bigger. Recognises counter shares as 4 twenty-fourths and 6 twenty-fourths respectively.</td><td>Suggests a reasonable understanding of halving and the implications of repeatedly halving, may not see part of a collection as a countable unit (e.g. group of 4 counters not seen as 1 four of 6 fours).
• Extend paper-folding to make and name parts in the halving, thirding and fifthing families (see above).
• Use discrete fraction models and sharing to ensure equal parts understood in this context (i.e. 4 counters seen as 1 of 6 equal parts).
• Use partitioning strategies to construct fraction diagrams and line representations for a broader range of fractions (see Partitioning paper in Additional Resources).
• Connect fraction diagrams to the region model of multiplication, e.g. three parts (thirds) by 4 parts (quarters) give 12 parts (twelfths) and more efficient mental strategies based on doubling and/or known facts.</td></tr>
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4.2 Fraction Naming Tool

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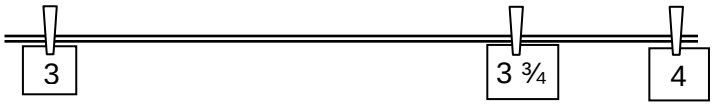
Observed Response	Interpretation/Suggested Teaching Response
Able to complete all tasks fairly confidently, determines number of parts in paper-folding task either by repeat halving or by using region representation (e.g. 8 rows of 8). Recognises counter shares as 1 sixth and 1 quarter respectively.	*Suggests that key ideas are well understood (i.e. number of parts/shares names the part and each part/share get smaller as total number of parts/shares increases).* • Use partitioning strategies to construct fraction diagrams and line representations for a broader range of fractions (see *Partitioning* paper in Additional Resources). • Draw attention to the key generalisation and the links to multiplication and division implicit in the process of paper folding and sharing. • Consider introducing/consolidating decimal fractions to hundredths by successively "*tenthing*". Use to compare, order, sequence and rename decimal fractions.

4.3 Fraction Making Tool

Although most Year 4 students are able to recognise and name simple fractional parts in relation to physical models (chocolate, pizzas, sandwiches etc), the introduction of more formal models (e.g. fraction diagrams and number lines) appears to be associated with a marked decline in student performance in this area. One of the possible reasons for this is that students do not have the strategies to generate their own, more formal fraction models and representations. Also, in the past, most of the practical work with fractions has focussed on proper fractions at the expense of mixed or improper fractions.

This task examines the extent to which students are able to generate their own fraction representations. It assumes some prior experience with practical models and a knowledge of fraction names.

Observed Response	Interpretation/Suggested Teaching Response
Can identify 1 half and possibly 1 third of the counters but not 5 eighths, may partition '*pizza*' and show 3 quarters but unable to complete much more beyond this.	*May not be familiar with non-unitary fractions or fraction s greater than 1 or have access to partitioning strategies beyond halving.* • Provide plenty of opportunities to develop non-unitary fraction language through 'real-world', informal fraction activities, labelling recognised parts and relating them to the whole, e.g. 3 *quarters* of the orange, 2 thirds of the netball court. • Extend work with fraction models to mixed fractions (e.g. 5 thirds, 2 and 3 quarters). • Consolidate the *halving* partitioning strategy using paper folding (to represent region diagrams) and a rope and pegs (to represent line segments, e.g. see below) and use this to support students' capacity to generate their own fraction models and diagrams (see *Partitioning* paper in Additional Resources).  • Consider introducing the *thirding* strategy (i.e. the thinking that 1 third is less than 1 half, estimate 1 third, leaving room for two more parts of the same size).

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4.3 Fraction Making Tool

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Observed Response	Interpretation/Suggested Teaching Response
Identifies 1 half and1 third of counters but may not recognise 5 eighths, partitions pizza and shows 3 quarters, able to make at least one mixed fraction model but experiences difficulty partitioning rectangle or number line on worksheet, may identify the whole for the tile task.	*Suggests some understanding of partitioning in relation to familiar objects and halving.* • Consolidate 'cutting' and sharing strategies using 'real-world' situations (e.g. cutting up sandwiches), make and name mixed fractions by identifying the unit. • Introduce/consolidate the *thirding* and *fifthing* partitioning strategies (see *Partitioning* paper in Additional Resources) via real-world models (see above) and use these to support students' capacity to generate their own fraction diagrams and number lines.
Able to complete most tasks reasonably well but may use make-all strategy to determine 5 eighths and additive strategies to partition region diagram and line.	*Suggests more developed understanding of partitioning but may be thinking additively instead of multiplicatively.* • Review *partitioning* strategies and the thinking associated with these (e.g. 1 fifth is less than 1 quarter, estimate 1 quarter, make 1 fifth slightly smaller than this, then halve remaining part and halve again). • Explore what happens when partitioning strategies are combined. Example: Halving then thirding produces sixths; Halving and then fifthing produces tenths. Link region diagram representations to region idea for multiplication and number line representations to "*for each*" idea for multiplication. • Practice making and naming an extended range of fraction representations (e.g. make sevenths by thinking 1 seventh is smaller than 1 sixth, I can estimate sixths by halving and thirding). • Consider introducing/consolidating decimal fraction representations to hundredths by successively "*tenthing*" region diagrams or number lines. Use to compare, order, sequence and rename decimal fractions.

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4.3 Fraction Making Tool

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Observed Response	Interpretation/Suggested Teaching Response
Completes all tasks confidently using known facts for counters tasks and multiplicative partitioning strategies based on halving (i.e. does not count from left to right, but divides area or line into equal parts).	*Suggests multiplicative basis for partitioning and a sound knowledge of how to construct fraction representations.* • Continue to explore what happens when partitioning strategies are combined (see above), link region diagram representations to "*region*" idea for multiplication and number line representations to "*for each*" idea for multiplication. • Consolidate and extend language and skills associated with partitioning, e.g. *Fraction Estimation* from Maths300 (Curriculum Corporation, 2001). • Consider introducing/consolidating decimal fraction representations to thousandths by successively "*tenthing*" region diagrams or number lines. Use to compare, order, sequence and rename decimal fractions.

4.4 Fraction Recording Tool

This task explores students' capacity to recognise and record common fractions and, in particular, their meanings for the numerator and denominator. This is important as some students in the middle years tend to treat these two numbers as discrete whole numbers (e.g. "*5 over 6*") that you "*do things to*", rather than seeing these as mutually dependent and indicative of a particular proportion or as a number in it's own right (the result of the dividing the numerator by the denominator).

Observed Response	Interpretation/Suggested Teaching Response
Little/no response or incorrect (e.g. says 5 halves or simply records 5 to indicate the number of pieces).	*May not understand task, suggests little/no appreciation of how fractions are made and named.* • Use paper-folding to make and name parts in the halving family (e.g. halves, quarters, eighths, sixteenths, thirty-seconds, sixty-fourths etc). • Focus on key generalisations regarding equal parts, number of parts and size of parts as number of parts increases (see Tools 4.1 and 4.2). • Use models and examples to distinguish between "*how many*" (numerator idea) and 'how much' (denominator idea). • Make a table that records the number of parts and the name of the parts. • Consider introducing the *thirding* strategy (see Partitioning paper in Additional Resources) using paper-folding, lengths of rope etc to make, name and count fractional parts to 1 and beyond.
Recognises 5 sixths but unsure about how to record or, if written, refers to 5 as 5 pieces and 6 as 6 pieces (i.e. little/no indication of relationship), may not recognise 2 thirds or be able to record mixed fraction, may recognise 6/15 in terms of a count of tiles.	*Suggests some understanding of how fractions are represented but a limited understanding of how they are recorded.* • Consolidate "*cutting*" and sharing strategies using "*real-world*" material and situations (e.g. cutting up sandwiches), make and name mixed fractions by identifying the unit. • Introduce/consolidate the *halving* and *thirding* partitioning strategies (see *Partitioning* paper in Additional Resources) via real-world models (see above) and use these to support students' capacity to generate their own fraction diagrams and number lines.

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4.4 Fraction Recording Tool

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Observed Response	Interpretation/Suggested Teaching Response
Recognises 5 sixths but unsure about how to record or, if written, refers to 5 as 5 pieces and 6 as 6 pieces (i.e. little/no indication of relationship), may not recognise 2 thirds or be able to record mixed fraction, may recognise 6/15 in terms of a count of tiles.	• Explore what happens when *halving* and *thirding* are combined, make, name and count fractional parts. • Provide plenty of opportunities to develop non-unitary fraction language through 'real-world', informal fraction activities, labelling recognised parts and relating them to the whole (e.g. 3 *quarters* of the orange, 2 thirds of the netball court). • Extend work with fraction models to mixed fractions (e.g. 5 thirds, 2 and 3 quarters), count forwards and backwards in fractional parts starting anywhere.
Explains symbol for 2 thirds (e.g. identifies 1 third as 2 sixths, shows 2), but may not be able to do this for 2 fifths, may recognise but not be able to record 2 and 3 eighths, records and justifies 6 fifteenths.	*Suggests some understanding of how fractions are recorded but may not have access to strategies to support fraction renaming.* • Use partitioning strategies, region diagrams and lines to explore and justify fraction renaming (see *Partitioning* paper in Additional Resources). • Introduce, model and discuss the generalisation: *if the total number of parts is increased/decreased by a certain factor then the number of parts required is increased/decreased by the same factor.* • Practice making, naming and recording a wide range of fraction representations.
Confidently completes most tasks, implicitly distinguishes between "*how many*" and "*how much*", and uses fraction names routinely.	*Suggests a reasonably thorough understanding of fraction recording and renaming.* • Consolidate and develop using a wide range of models and representations including mixed and improper fractions. • Consider introducing written procedures for adding and subtracting, like and related fractions (see Booker et al, 2003). • Consider introducing decimal notation, tenths as a new place-value part (see Booker et al, 2003).

4.5 Decimal Fraction Naming and Recording Tool

There is a considerable research evidence to suggest that decimal numeration is not well understood at this level. One of the reasons for this is that decimals are often introduced before students are ready using inappropriate models such as money and MAB base 10 materials. Before working with decimals students need a solid understanding of common fractions, in particular how parts are formed and named. Tenths can be introduced as a new place-value part (e.g. see Booker et al, 2003) once students have access to appropriate partitioning strategies (e.g. *halving* and *fifthing*, see *Partitioning* paper in Additional Resources) and are able to construct their own fraction diagrams and line representations.

This task examines the extent to which students can recognise and record decimal fractions. It assumes that students have a reasonable understanding of common fractions and have been introduced to decimal recording.

Observed Response	Interpretation/Suggested Teaching Response
Measures streamer in terms of centimetres, may not be able to record in metres or recognise what 4 means in 1.45, may recognise and record 6 tenths.	*Suggests a fairly limited understanding of what decimals represent and how they might be used and recorded.* • Use a range of measurement activities to explore tenths in the real world, review relationship between different units (e.g. 1 cm is 10 mm, 1 metre is 100 cm). • Use partitioning strategies (see *Partitioning* paper in Additional Resources) to make and name models of ones and tenths (e.g. 3 ones and 7 tenths), record using common fractions and decimal notation. • Review tenths as a place-value part and the idea more generally that 1 tenth of these is one of those, consolidate by providing opportunities to compare, order, sequence, count forwards and backwards in place-value parts, and rename.
Explains and justifies 4 in 1.45 metres, recognises and records tenths (for diagram and green line) but may experience some difficulty with hundredths and/or per cent.	*Suggests a reasonable understanding of how decimals are used to measure length but more general understanding of decimals may be limited to tenths.* • Extend activities with measurement to explore hundredths and thousandths and establish relationship between different units (e.g. 1000 grams is 1 kilogram).

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4.5 Decimal Fraction Naming and Recording Tool

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Observed Response	Interpretation/Suggested Teaching Response
Explains and justifies 4 in 1.45 metres, recognises and records tenths (for diagram and green line) but may experience some difficulty with hundredths and/or per cent.	• Use per cent 'benchmarks' to support hundredths (e.g. 50 per cent means 50 out of every hundred, 50 hundredths). • Use partitioning strategies (see *Partitioning* paper in Additional Resources) to make and name models of ones, tenths and hundredths (e.g. 2 ones, 8 tenths and 7 hundredths), link to 'for each' idea (i.e. each tenth is divided into 10 parts, so 10x10 parts altogether), record using common fractions and decimal notation. • Review hundredths as a place-value part and the idea more generally that 1 tenth of these is one of those, consolidate (see above).
Completes all tasks fairly confidently.	*Suggests a reasonably sound understanding of how decimal parts to hundredths are made, named and recorded.* • Continue to make, name and record decimal fractions, consolidate by providing opportunities to compare, order, sequence, count forwards and backwards in place-value parts, and rename (see *Developing the Big Ideas in Number* paper in Additional Resources). • Consider extending decimal place-value to thousandths and using the idea of a magnifying glass (see *Partitioning* paper in Additional Resources) to locate thousandths (e.g. 3.582).

4.6 Comparing and Ordering Tool

Comparing and ordering fractions and decimals requires a sound understanding of how these numbers are represented, named, recorded, and renamed. Far too many students rely on inappropriate rules and procedures to interpret and rename these numbers which results in misconceptions such as the larger the denominator, the larger the fraction, the more digits after the decimal point the larger (or smaller) the number is and so on.

This task examines the extent to which students have a sense of the relative magnitude of fractions and can use this to estimate and make comparative judgements. It also examines the extent to which students understand how decimal numbers can be renamed and their strategies for ordering decimal numbers.

Observed Response	Interpretation/Suggested Teaching Response
May recognise or guess which fraction is larger (3 quarters) or smaller (48/80) but unable to say why, may choose inappropriate fractions for Fraction Mat task, generally unable to order decimal cards.	*Suggests fractions and decimals not well understood in terms of how they are represented, named, recorded, and/or renamed.* • Check the extent to which students can make, name and record common fractions and decimals (see Tools 4.1 to 4.5). • Model and practice sequencing common fractions and decimals using a rope and pegs, play appropriate games, e.g. Make a Whole (see Additional Resources). • Use partitioning strategies (see *Partitioning* paper in Additional Resources) to make and name models of ones and tenths (e.g. 3 ones and 7 tenths), record using common fractions and decimal notation. • Review tenths as a place-value part and the idea more generally that 1 tenth of these is one of those, consolidate by providing opportunities to compare, order, sequence, count forwards and backwards in place-value parts, and rename. • Consolidate fraction renaming using partitioning strategies (see *Partitioning* paper in Additional resources).

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4.6 Comparing and Ordering Tool

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Observed Response	Interpretation/Suggested Teaching Response
Identifies and weakly justifies (e.g. based on rough estimation) larger and smaller fraction, makes a reasonable choice for Fraction Mat task (e.g. 1 quarter and 3 fifths), and correctly orders some of the Decimal Cards.	*Suggests some understanding of how fractions and decimals are represented, named, recorded, and/or renamed.* • Check the extent to which students can make, name and record an extended range of common fractions and decimals, including mixed fractions. • Consolidate the use of partitioning strategies to make and rename fractions and decimals to hundredths (see *Partitioning* paper in Additional resources).
Identifies and strongly justifies (i.e. renames) larger and smaller fraction, makes a closely reasoned choice for Fraction Mat task (e.g. 3 quarters and 1 fifth or 5 sixths and 1 seventh), and correctly orders most of the Decimal Cards.	*Suggests relatively sound understanding of how fractions and decimals are represented, named, recorded, and/or renamed.* • Extend comparing, ordering, sequencing and renaming activity to include more complex common fractions and decimals to thousandths. • Consider introducing/consolidating operations on common fractions and decimals (e.g. addition, subtraction and multiplication).

4.7 Rational Number Sequencing Tool

This task should only be used where students have demonstrated some facility with fractions and decimals and have some appreciation of where negative numbers live. Student responses to this task indicate the extent to which they understand rational number recording and can compare, order/sequence, and rename common fractions and decimals. As such it is quite a powerful summative assessment tool.

Observed Response	Interpretation/Suggested Teaching Response
Experiences difficulty locating numbers and may attempt to locate recognised numbers by counting from the left or guessing.	*Suggests rational number not well understood.* • Check the extent to which students can make, name and record common fractions and decimals (see Tools 4.1 to 4.5). • Consolidate a sense of negative numbers and where they "*live*" using real-world models (e.g. temperature scales). • Model and practice sequencing common fractions and decimals using a rope and pegs, play appropriate games, e.g. adapt Place-Value Game (see Additional Resources) to sequence hundredths.
Locates some cards (e.g., 0.2 and 1.5) reasonably accurately, but may use fraction as operator idea to locate ¾ and 50% (e.g. ¾ of the rope), and/or experience some difficulty with negative number cards.	*Suggests some confusion between fraction as operator and fraction as number.* • Explore fraction renaming strategies (see *Partitioning* paper in Additional resources). • Introduce/review link between fractions and division, in particular, the notion that m/n means m divided by n ($n \neq 0$), use to rename fractions as decimals and per cents. • Provide opportunities to practice the use partitioning strategies to locate fractions, decimals, and negative numbers.
Able to locate most cards reasonably accurately.	*Suggests sound understanding of rational numbers and access to appropriate partitioning and renaming strategies.* • Extend sequencing activity to include more complex rational numbers. • Consider introducing/consolidating operations on common fractions and decimals (e.g. addition, subtraction and multiplication). • Consider introducing irrational numbers.