Place Value Tools

Common Misunderstanding

Many students who are able to identify place-value parts (e.g. they can say that there are 4 hundreds 6 tens and 8 ones in 468) and count orally to 100 and beyond; still think about or imagine 2 and/or 3 digit collections additively in terms of ones (i.e. 468 is actually understood as the sum of 400 ones, 60 ones and 8 ones).

This could be due to/associated with:

- inadequate part-part-whole knowledge for the numbers 0 to 10 and/or an inability to trust the count (see Trusting the Count Tools);
- an inability to recognise 2, 5 and 10 as composite or countable units (often indicated by an inability to count large collections efficiently);
- little or no sense of numbers beyond 10 (e.g. fourteen is 10 and 4 more); and/or;
- a failure to recognise the structural basis for recording 2 digit numbers (e.g. sees and reads 64 as "*sixty-four*", but thinks of this as 60 and 4 without recognising the significance of the 6 as a count of tens, even though they may be able to say how many tens in the tens place).

Students need a deep understanding of the place-value pattern, *10 of these is 1 of those*, to support more efficient ways of working with 2 digit numbers and beyond. Place-value is difficult to teach and learn as it is often masked by successful performance on superficial tasks such as counting by ones on a 0-99 or 1 - 100 Number Chart. The structure of the base ten number system is essentially multiplicative, as it involves counts of different sized groups that are powers of 10.

Unfortunately, place-value is often introduced before students have demonstrated an understanding that the numbers 2 to 10 can be used as countable units and/or before any informal work with equal groups.

As a consequence, many students develop misconceptions in this area which serve to undermine their capacity to use place-value based strategies to support efficient mental and written computation and their later understanding of larger whole numbers and decimal fractions.

A key indicator of the extent to which students have developed a sound basis for place-value is the extent to which they can efficiently count large collections and confidently make, name, record, compare, order, sequence, count forwards and backwards in place-value parts, and rename 2 and 3 digit numbers in terms of their parts.

2.1 Number Naming Tool

MATERIALS

- 26 counters in a suitable jar or container.
- 7 bundles of ten icy pole sticks or straws and 22 single sticks or straws.
- A 0-99 Number Chart and a Masking Card (see Place Value Resources).

INSTRUCTIONS

Bold type indicates what should be said.

Empty container of counters in front of student.

Say: **"Can you count these as quickly as possible and write down the number please?"** Note how the count is organised and what is recorded.

If not 26, ask:

"Are you sure about that? How could you check?"

Once student has recorded 26, circle the 6 in 26 ask:

"Does this (point to the 6) **have anything to do with how many counters you have there?"** Indicate the collection. Note student's response. Circle the 2 in 26 and repeat the question. Note student's response. Place counters back in the container.

Place bundles and sticks in front of the student and ask:

"Can you make 34 using these materials please?" Note student's response. If student asks or moves to unbundle a ten…

Say: **"Before you do that, is there any way you could use these** (pointing to the bundles of ten) **to make 34?"** Note student's response. Remove sticks.

Tip out the container of 26 counters and ask student to count these again and record the number. Note response.

Ask: **"Can you put these into groups of four please?"** Once this is completed, point to the 26 that has been recorded and circle the 6.

Ask: **"Does this have anything to do with how many counters you have?"** Circle the 2 in 26 and repeat the question. Note student responses.

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2.1 Number Naming Tool

Continued from previous page

If counter task handled reasonably well, place the 0-99 Number Chart in front of the student. Cover the numbers 33 to 66 with the Masking Card.

Ask: **"Can you count on by ones from 41 please?"** If done easily, point to 57.

Ask: **"Can you count back by ones from here please?"** Note student's response.

Run your finger down the column headed by 5.

Ask: **"What are we counting on by now?"** If student says "*fives*", remove mask and try to find out what he/she is thinking by asking:

"Do you still think we are counting by fives … Why?"

Proceed to next question. If student answers "*ten or tens*" to initial question, leave mask in place and ask:

"Can you count on from 15 by tens until I say 'stop'?" If student stops at 95, encourage him/her to continue. Say "*stop*" when student no longer continues or at 135 (whichever comes sooner). Note response and whether or not student can proceed beyond 95.

2.2 Efficient Counting Tool

MATERIALS

- 56 stackable counters.
- A container to hold counters.
- 13 bundles of ten icy pole sticks and 16 single sticks/straws.
- Paper and a pencil/pen.

INSTRUCTIONS

Bold type indicates what should be said.

Place container of counters in front of the student.

Say: **"I'm going to tip all these counters out and I would like you to count them as quickly as possible … Ready? …** Tip out counters. "**Go!"**

Note how student counts and how he/she organises the count (e.g. counts by ones, counts by twos, systematically moves counters to avoid recounting or groups to make count easier).

If student counts quickly and accurately by whatever method, ask…

"How would you write that number? ... Why would you write it like that? … If I counted that collection, would I get the same number?" Note student's response.

If student appears to lack confidence, counts relatively slowly by ones and/or counts inaccurately. Stop them about half-way.

Say: **"That's going to take a long time isn't it? Let's put that amount over here. Let's see if we can count what's left in a quicker way."** Model counting by twos to 14.

Say: **"If I do this, will this work? …"** Then ask student to continue from 14.

Say: **"Can you keep on counting like this?** Note accuracy and speed. Stop and proceed to next task if, after one attempt to self correct, student still counts incorrectly. Otherwise, ask:

"How would you write that number? ... Why would you write it like that? … If I counted that collection, would I get the same number?"

Note student's strategy and written response.

Place all the icy-pole sticks in front of the student. Pick up one bundle of ten and ask…

"How many sticks are in this bundle?" If the student guesses or appears uncertain, unbundle and ask him/her to count before proceeding. If the student says "*ten*" fairly confidently…

Say: **"Okay, can you count these for me and tell me how many please? … How would you write that? … Why would you write it like that?"**

Note student's strategy and written response.

2.3 Sequencing Tool

MATERIALS

- A length of rope (approximately 1 metre).
- Some clothes pegs.
- Sequencing Cards (see Place Value Resources).

INSTRUCTIONS

Bold type indicates what should be said.

Stretch out rope in front of the student (anchor ends if necessary).

Say: **"Let's imagine all of the numbers from 0 to 100 are on this rope."** As you say this, peg the "*0*" card at the beginning of the rope and the "*100*" card at the end of the rope.

Place the "*48*" card in front of the student.

Say: **"Can you peg this card on to the rope to show where you think that number would be? Can you tell me why you put it there?"** Note where the card is placed and student's response/strategies.

Repeat with the 67and 26 card. Note responses and strategies.

If hesitant or unable to proceed at any point, remove the 100 card and replace it by the 20 card.

Say: **"Okay, now let's imagine all of the numbers from 0 to 20 are on this rope."**

Place the "*8*" card in front of the student.

Say: **"Can you peg this card on to the rope to show where you think that number would be? Can you tell me why you put it there?"** Note student's response/strategies.

If done reasonably well, place the 16 card in front of student and ask them to peg that on the rope as well. Note student's strategies.

2.4 Renaming and Counting Tool

MATERIALS

- 3 MAB hundreds, 17 MAB tens and 6 MAB ones.
- Renaming and Counting Cards (see Place Value Resources).
- Paper and pen/pencil.

INSTRUCTIONS

Bold type indicates what should be said.

Place 3 hundreds, 17 tens and 6 ones in front of the student. Make sure student understands that they are hundreds, tens, and ones.

Ask: **"Can you write down the number shown by these blocks please?"** Note student's response. If number is incorrect, ask student to explain their answer then move on to the Card A task.

If correct, that is, 476 is recorded, remove materials to one side.

Ask: **"If you could only use tens and ones to make that number, how many tens would you need?"** Note student's response and/or strategies. If student appears hesitant, try to find out what he/she is thinking by asking:

"Can you tell me what you are thinking about?" or

"What are you trying to do?"

Place Card A in front of the student.

Ask: **"Can you read that number please?"** Note response, and then ask:

"Can you think of a number that is smaller than this but larger than 50?" Note student's response, then point to Card A again and ask:

"Now, can you write down a number that is 2 tens larger than this?" Note student's response. If student experiences some difficulty with this, try to find out why. Either stop at this point or proceed to Card B if appropriate.

Present Card B and ask:

"Can you read that number please?" Note response, and then ask:

"Can you think of a number that is smaller than this but bigger than 517?" Note response. If correct, ask:

"This time, can you write down a number that is at least 1 hundred more than this but smaller then 968?" Note response. If student hesitates, remove card and proceed to Card C.

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2.4 Renaming and Counting Tool

Continued from previous page

Present Card C.

Ask: **"Can you read that number for me please?"** If correct, ask:

"Can you count on by ones from that number please? If student counts on by ones fairly easily, point to Card C again.

Say: **"Thank you, now what if I asked you to count backwards by tens. What number do you think would be ten less than this number?"** Note student's response, explore their thinking if appropriate.

Place Value Advice

2.1 Number Naming Tool

This tool has four different elements, the kidney bean task, the bundling sticks task, the regrouping task, and the number chart task. The observations and advice associated with each task are presented in turn below.

COUNTERS

Student responses to this task indicate the meanings they attach to 2-digit numerals. A version of this task was originally employed by Ross (1989)[1] who identified five stages in the development of a sound understanding of place-value, each of which appears in some form in the advice below.

Observed Response	Interpretation/Suggested Teaching Response
Little/no response	*May not understand task.* • Repeat at a later date.
Response given but not indicative of strong place-value knowledge, e.g., refers to 6 ones or physical arrangement such as "*2 groups of 3*" for circled 6, and "*twenty*" for circled 2.	*Suggests 26 is understood in terms of ones, or 20 (ones) and 6 ones, may not trust the count of 10 or see 2 as a count of tens.* • Check extent to which child trusts the count for 10 by counting large collections (see Tool 2.2). • Practice *making, naming* and *recording* tens and ones, emphasising the count of tens in the tens place and the count of ones in the ones place (see Booker et al.[2] for further details).

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[1] Adapted from a task used by Ross (1989). Parts, wholes, and place-value: A developmental view. *Arithmetic Teacher, 36*(6). 47-52.
[2] Booker, G., Bond, D., Sparrow, P. & Swan, L. (2003). *Teachng Primary Mathematics*. Melbourne: Pearson-Prentice Hall.

2.1 Number Naming Tool

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COUNTERS

Observed Response	Interpretation/Suggested Teaching Response
Says 6 ones and 2 tens fairly quickly.	*Appears to understand the basis on which 2-digit numbers are recorded.* • Consolidate 2-digit place-value by *comparing* two numbers presented in different ways (e.g. 3 bundles of ten and 17 ones and a card showing 46, which is bigger/smaller? Why/how do you know?), *ordering/sequencing* (order 5 or more 2-digit numbers or place in sequence on a rope from 0 to 100, discuss and refine strategies), *counting forwards and backwards in place-value parts* starting anywhere (e.g. 27, 37, 47 (clap), 46, 45, 44, 43, …), and by *renaming* (e.g. 45 is 4 tens and 5 ones or 45 ones, which is easier to see?). • Consider introducing 3-digit place-value (see Booker et al. for further details).

BUNDLING STICKS

Student responses to this task indicate their understanding of place-value and the extent to which they trust the count of 10, that is, they can treat 10 as a countable unit.

Observed Response	Interpretation/Suggested Teaching Response
Little/no response, incorrect or insists on using/counting by ones only.	*May not understand task, does not trust the count of 10.* • Check extent to which child *trusts the count* for 10 by counting large collections (see Tool 2.2) and review *subitising* and *part-part-whole ideas* for 10 (see Trusting the Count Advice). • Practice *making, naming* and *recording* tens and ones, emphasising the count of tens in the tens place and the count of ones in the ones place (see Booker et al. for further details).

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2.1 Number Naming Tool

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BUNDLING STICKS

Observed Response	Interpretation/Suggested Teaching Response
Counts ones, attempts or asks to unbundle tens in order to continue count of ones, after prompt may check count of bundle then uses either 2 tens and 14 ones or 3 tens and 4 ones.	*Suggests place-value ideas not well established, may not trust the count of 10 if student checks the number in a bundle.* • Check *trust the count*, review *subitising* and *part-part-whole ideas* for 10 and *making, naming* and *recording* tens and ones (see Trusting the Count Advice). • Consolidate 2-digit place-value by *comparing* 2 numbers (materials, words and symbols), *ordering/sequencing* (by ordering 5 or more 2-digit numbers or placing in sequence on a rope from 0 to 100), *counting forwards and backwards in place-value parts* starting anywhere (e.g., 27, 37, 47 (clap), 46, 45, 44, 43, …), and by *renaming* (e.g. 45 is 4 tens and 5 ones or 45 ones).
Uses 3 tens and 4 ones to make 34 without any hesitation.	*Suggests sound understanding of place-value, particularly if combined with ability to recognise 6 as 6 ones and 2 as 2 tens in 26.* • Consolidate 2-digit place-value by *comparing* two numbers presented in different ways (e.g. 3 MAB tens and 17 ones and a card showing 46, which is bigger/smaller? Why/how do you know?), *ordering/sequencing* (order 5 or more 2-digit numbers or place in sequence on a rope from 0 to 100, discuss and refine strategies), *counting forwards and backwards in place-value parts* starting anywhere (e.g. 27, 37, 47 (clap), 46, 45, 44, 43, …), and by *renaming* (e.g. 45 is 4 tens and 5 ones or 45 ones, which is easier to see?). • Consider introducing 3-digit place-value (see Booker et al. for further details).

2.1 Number Naming Tool

GROUPING TASK

Student responses to this task indicate the strength of their understanding of place-value by exploring the extent to which they can be distracted by the regrouping and the perceptual image it presents (6 groups of 4 and 2 ones remaining). Interestingly, some students who referred to the 2 in 26 as "*twenty*" in the first instance are prompted to refer to the 2 in 26 as "*2 tens*" after the grouping exercise.

Observed Response	Interpretation/Suggested Teaching Response
Little/no response or refers to 6 as the number of groups of 4 and 2 as the 2 remaining ones.	Distracted by the visual arrangement to override whatever else they may know about what "*26*" means, suggests little/no place-value knowledge. *May not understand task, does not trust the count of 10.* • Check extent to which child *trusts the count* for 10 by counting large collections (see Tool 2.2) and review *subitising* and *part-part-whole ideas* for 10 (see Trusting the Count Advice). • Practice *making, naming* and *recording* tens and ones, emphasising the count of tens in the tens place and the count of ones in the ones place (see Booker et al. for further details).
Is not distracted by visual image or regrouping, but refers to 2 as "*twenty*".	*Suggests place-value ideas not well established, may not trust the count of 10, and suggests thinking based on ones, not a count of tens and a count of ones.* • Check *trust the count*, review *subitising* and *part-part-whole ideas* for 10 and *making, naming* and *recording* tens and ones (see above). • Consolidate the count of tens using patterns on a 0-99 Number Chart, discuss what remains the same, what is different, why? Describe pattern in place-value terms (e.g., "going up/down in the tens by 1' rather than 'adding/subtracting 10").

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2.1 Number Naming Tool

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GROUPING TASK

Observed Response	Interpretation/Suggested Teaching Response
Says 6 ones and 2 tens fairly quickly.	*Appears to understand the basis on which 2-digit numbers are recorded.* • Consolidate 2-digit place-value by *comparing* two numbers presented in different ways (e.g. 3 MAB tens and 17 ones and a card showing 46, which is bigger/smaller? Why/how do you know?), *ordering/sequencing* (order 5 or more 2-digit numbers or place in sequence on a rope from 0 to 100, discuss and refine strategies), *counting forwards and backwards in place-value parts* starting anywhere (e.g. 27, 37, 47 (clap), 46, 45, 44, 43, …), and by *renaming* (e.g. 45 is 4 tens and 5 ones or 45 ones, which is easier to see?). • Consider introducing 3-digit place-value (see Booker et al. for further details).

NUMBER CHART TASK

Student responses to this task indicate their understanding of the place-value pattern and the extent to which they recognise the count of 10 and treat 10 as a countable unit.

Observed Response	Interpretation/Suggested Teaching Response
Little/no response or hesitant to count hidden numbers on from 41 or back from 57.	*Unlikely response but may not understand task, needs to see the numbers to generate the count, and/or is unsure about the number naming sequence.* • Check extent to which child can accurately count a physical collection (see Tool 2.2). • Practice *making, naming* and *recording* tens and ones, emphasising the count of tens in the tens place and the count of ones in the ones place (see Booker et al. for further details).

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2.1 Number Naming Tool

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NUMBER CHART TASK

Observed Response	Interpretation/Suggested Teaching Response
Little/no response or hesitant to count hidden numbers on from 41 or back from 57.	• Draw attention to the patterns inherent in the 0-99 Number Chart by talking about the 1 ten family, 2 ten family, 3 ten family etc and how each number is represented, e.g. 34 is in the 3 ten family, it is made up of 3 tens and 4 ones. Discuss what remains the same and what changes in each row (tens remain the same, ones change). • Rehearse oral counting sequence and counting larger collections efficiently (see Tool 2.2).
Manages to count on/back by ones fairly easily but says "fives" when asked to identify column count. May count on by tens once mask removed.	*May not understand task but more likely to be distracted by visual perception suggesting understanding of place-value pattern not very robust, may only understand count of tens in terms of multiplies of ten and ones (i.e. 10, 20, 30, 40, …)*. • Practice *making, naming* and *recording* tens and ones, emphasising the count of tens in the tens place and the count of ones in the ones place (see Booker et al. for further details). • Draw attention to the patterns inherent in the 0-99 Number Chart (see above). Discuss what remains the same and what changes in each row (tens remain the same, ones change) as well as each column (ones remain the same, tens change). • Practice counting on/back in place-value parts starting from anywhere using Number Charts that extend beyond 100.

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2.1 Number Naming Tool

Continued from previous page

NUMBER CHART TASK

Observed Response	Interpretation/Suggested Teaching Response
Recognises count of tens and can count on by tens to 95 but hesitates or counts on by ones or fives to arrive at 105.	*May not appreciate that tens can be counted as countable units in the same way that any other "object" might be counted, or that numbers can be renamed in terms of their place-value parts.* • Practice *renaming* hundreds, tens and ones, using MAB materials (e.g. 124 can be shown as 1 hundred 2 tens and 4 ones, as 12 tens and 4 ones, or as 124 ones). Record in words as in example given. Discuss which is the easiest way to remember or think about these numbers. • Practice counting on/back in place-value parts starting from anywhere using Number Charts that extend beyond 100.
Recognises count of tens and counts to 135 with little difficulty.	*Suggests a sound understanding of the basis on which 2-digit numbers are recorded.* • Introduce 3-digit place-value (see Booker et al. for further details). • Use MAB and Number Expanders to consolidate 3-digit place-value by *comparing* 2 numbers (materials, words and symbols), *ordering/sequencing* (by ordering 5 or more 3-digit numbers or placing in sequence on a rope from 0 to 1000), *counting forwards and backwards in place-value parts* starting anywhere. Example: 327, 337, 347 (clap), 346, 345, 344, 343, … and by *renaming* (e.g. 845 is 8 hundreds 4 tens and 5 ones, 84 tens and 5 ones, or 845 ones). • Use Number Chart activities (see Additional Resources) to reinforce counting patterns.

2.2 Efficient Counting Tool

This tool has two different elements, the stackable counters task and a bundling sticks task. The observations and advice associated with each task are presented in turn below.

STACKABLE COUNTERS

Student responses to this task indicate the extent to which they can use grouping methods to count a large collection efficiently (e.g., by twos, fives or tens). This also provides an indication of the extent to which students *trust the count* of the particular group chosen and see the group as a countable unit in its own right which is an important pre-requisite for working with multiplication later on. Consider the student's first response and whether or not they can adopt a more sophisticated strategy after the modelling prompt.

Observed Response	Interpretation/Suggested Teaching Response
Persists with counting by ones despite prompt.	*May not trust the count for twos or fives and/or may not be familiar enough with counting word sequences involved (e.g. 2, 4, 6, 8 … or 5, 10, 15, …).* • Use 0-99 Number Chart to practice counting word sequences for twos, fives and tens. • Model and practice counting large collections. Point out the inefficiency and inaccuracy of counting by ones. Establish the value of counting by twos and fives. *Chicken Scramble* is a purposeful counting activity which involves placing a large amount of counters (enough for about 40-50 per student, the "*chicken feed*") in the middle of a group of students (the "*chickens*"). Students are advised not to be greedy chickens then, on the word, "*go*", students collect their share of the food. Before counting, students are asked if they think anyone has been greedy (amounts can be moved around accordingly), then the "*chickens*" are asked to count their "*food*" to see how fair their share was.

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2.2 Efficient Counting Tool

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STACKABLE COUNTERS

Observed Response	Interpretation/Suggested Teaching Response
Starts counting by ones but after prompt counts by twos, fives or tens, or starts counting by twos, fives or tens but experiences some difficulty with counting word sequence and/or with any remaining counters.	*May not completely trust the count for the group size involved or recognise that counting by a given group still tells how many. This is suggested where students get 'locked in' to the counting sequence to the point where they cannot deal with the 1 remaining (if counting by fives) or the 6 remaining (if counting by tens).* • Model and practice counting large collections (see above). • Use a 0-99 Number Chart to count by twos, fives and tens, stopping periodically to model *counting on from*, e.g. "*… 55, and 1 more?*", or "*… 60 and 7 more?*"
Counts fluently by fives or tens and adds on any remaining ones.	*Able to deal with composite units, and trusts the result of the count, ready to move to formal place-value.* • Introduce 2-digit place-value by making and counting tens as countable units, e.g., use bundling materials such as icy-pole sticks or straws to make tens and count as 1 ten, 2 tens, 3 tens etc. • For remaining steps see Booker et al (2003).

2.2 Efficient Counting Tool

COUNT BUNDLING STICKS

Student responses to this task indicate the extent to which they can treat ten as a countable unit, count in tens and ones and rename in terms of place-value parts.

Observed Response	Interpretation/Suggested Teaching Response
Treats bundles of tens as ones and counts all as ones despite prompt.	*Can count using one-to-one correspondence but does not trust the count of 10.* • Work on *subitising* and *part-part-whole knowledge* for numbers 1-5 then use this to build knowledge of numbers 6-10 as mental objects (evident when these numbers can be recognised using subitising and part-part-whole knowledge). • Once *part-part-whole knowledge* established for numbers 1-10, use Ten-frames and *Open Number Lines* (see Level 1 Advice) to build a sense of numbers beyond 10. • Introduce 2-digit place-value by making and counting tens as countable units, e.g. use bundling materials such as icy-pole sticks or straws to make tens and count as 1 ten, 2 tens, 3 tens etc. • For remaining steps see Booker et al (2003).
Recognises bundles as tens, partially counts by tens (e.g., may stop at 100, then proceed by ones), may experience some difficulty with counting word sequence.	*May not be familiar enough with counting word sequence for larger numbers, may not realise that a count of tens can go beyond 100.* • Use extended Number chart (e.g. 60-159) to practice counting forwards and backwards in ones and tens to strengthen number words and make patterns explicit.
Says and records 146 by counting tens and ones systematically.	*Appears to understand how tens and ones are represented, counted, named and recorded.* • Consolidate 2-digit place-value by *comparing* 2 numbers (materials, words and symbols), *ordering/sequencing* (by ordering 5 or more 2-digit numbers or placing in sequence on a rope from 0 to 100), *counting forwards and backwards in place-value parts* starting anywhere (e.g. 27, 37, 47 (clap), 46, 45, 44, 43, …), and by *renaming* (e.g. 45 is 4 tens and 5 ones or 45 ones).

2.3 Sequencing Tool

This task involves partitioning and should only be used where students have demonstrated a good grasp of 2-digit place –value and have some appreciation of halving. Student responses to this task indicate the extent to which students can locate a 2-digit number in relation to a given range of numbers. This is an important aspect of number sense (proportion) and underpins later work with division and fractions.

Partitioning at this level is a form of visual division. In this case, it is evident if students use their knowledge of halves and halving to make an informed (usually reasonably accurate) judgement about where to locate 48 ("*it's about half*") and 26 ("*It's just a bit more than a quarter*"). For 67, students may know that this "*is about 2 thirds*", but they are more likely to reason on the basis of what they know about halves and quarters in relation to 100, e.g. "*it's between a 50 and 75 but closer to 75*".

Observed Response	Interpretation/Suggested Teaching Response
Some difficulty locating numbers larger than 20 but reasonable attempt for numbers less than 20 on the 0 to 20 rope, may attempt to locate or justify placements by counting intervals from 0 using card width as a measure.	*Suggests numbers beyond 20 not well understood in terms of relative magnitude, possibly seen only as count of ones* • Consolidate 2-digit place-value by making, naming, recording, comparing etc (see above). • Model and practice ordering and sequencing 2-digit numbers, e.g. Place-Value Game (see Additional Resources).
Numbers larger than 20 placed more or less correctly, but actions and/or reasons given suggest counting rather than halving or partitioning strategies.	*Suggests numbers understood additively, that is, as a count from left to right, may not see interval marked by 0 to 100 as something that can be partitioned to locate numbers* • Review and discuss every-day halving, e.g., halving an orange, a length of paper tape, a piece of paper etc, • Review *doubling and halving*, discuss numbers in terms of their relationship to other numbers, e.g., 10 is half of 20, 30 is half of 60 and so on, demonstrate in class using a 3-4 metre length of rope, number cards and pegs.
Cards placed fairly accurately with relatively little hesitation, explanations based on partitioning, e.g. halving and/or fraction fact knowledge.	*Suggests sound knowledge of relative magnitude of 2-digit numbers in relation to 100 and basic fraction fact knowledge of halves and halving.* • Make the *halving* strategy more explicit by using a range of materials such as coloured square paper, paper streamers, counters etc and discussing the implications of successive halving. • Consider introducing the *thirding* and *fifthing* partitioning strategies (see Partitioning Paper in the Additional Resources).

2.4 Renaming and Counting Tool

This tool has two different elements, the MAB task and a card-based task. The observations and advice associated with each task are presented in turn below.

MAB MATERIALS

This task would only be used with students who had demonstrated a sound understanding of 2-digit place-value (see Tool 2.1) and are familiar with the MAB materials. Student responses to this task indicate the extent to which they understand 3 and 4-digit place-value.

Observed Response	Interpretation/Suggested Teaching Response
Little/no response, counts actual blocks (26), or says something like "*3 flats, 17 longs and 6 minis*".	*May not understand task and/or is unfamiliar with MAB materials, does not recognise significance of units.* • Review 2-digit place-value knowledge (see Advice for Tool 2.1 above).
Number recorded does not indicate that tens need to be regrouped as hundreds, e.g., says "*3 hundreds, 17 tens and 6 ones*", or records 3176 or 376.	*Suggests hundreds not understood as 10 tens, may not fully appreciate the role of position in recording numbers.* • Review and consolidate 2-digit place-value (see above). • Play *Trading Game* to given 3 digit numbers to reinforce that10 tens is 1 hundred (see Booker et al, 2003).
Number recorded correctly as 476, but unable to say how many tens would be needed, may attempt to count the number of tens by counting all (e.g., 10, 20, 30 …) and keeping track of the count or by formally dividing 476 by 10.	*May not appreciate that numbers can be renamed in terms of place-value parts in a variety of ways, may interpret task in terms of quotition division (how many tens in 476?).* • Review and consolidate 2 and 3-digit numbers, that is, make, name, record, compare, order/sequence, count forwards and backwards in place-value parts and rename numbers in more than one way (see Booker et al, 2003). • Model and practice renaming in particular using MAB and Number Expanders.
476 recorded, indicates 47 tens needed fairly quickly or by counting tens in a way that shows an understanding of 10 tens in 1 hundred (e.g., "10 tens in 1 hundred so 40 tens and 7 more.	*Demonstrates a relatively sound understanding of 3-digit place-value.* • Consolidate 3-digit place-value (see above). • Consider introducing or review and consolidate 4-digit numeration as appropriate (see Booker et al, 2003).

CARD TASKS

Observed Response	Interpretation/Suggested Teaching Response
Reads 86 correctly, provides an appropriate smaller number for Card A, may or may not provide a number that is 2 tens larger for Card A and experiences some difficulty with Card B.	*Suggests 2 and/or 3–digit number knowledge not very well established.* • Review and consolidate 2-digit place-value knowledge (Advice for Tools 2.1 above). • Consider introducing and/or reviewing and consolidating 3-digit place-value as appropriate.
Able to provide appropriate numbers for Cards A and B, but tends to provide numbers such as 520, 600 or 670 for a number smaller than 673 and 800 or 900 for a number at least 1 hundred more than 673 but smaller than 968, may not be able to read 5308 or count on by ones from 5308.	*Suggests 3 and/or 4-digit place-value knowledge not very well established.* • Review and consolidate 3-digit place-value knowledge (see above). • Consider introducing and/or consolidating 4-digit place-value as appropriate (see Booker et al, 2003).
Able to read and count on by ones from 5308, may have difficulty counting back by tens.	*Suggests 4-digit place-value knowledge not very well established, may not be aware that renaming can be used to support counting back by tens* • Consider introducing and/or consolidating 4-digit place-value as appropriate (see Booker et al, 2003) • Illustrate the use of renaming in situations like this by using a Number Expander. Example: Show 5308 as 5 thousands 30 tens and 8 ones This facilitates counting backwards in tens as 1 ten is taken from 30 to give 5 thousands 29 tens 8 ones or 5298.
Identifies 5298 as 1 ten less than 5308.	*Suggests a sound understanding of 4-digit place-value.* • Consider introducing and/or consolidating 5-digit place-value as appropriate.