Trusting the Count Tools

Common Misunderstanding

Many students who are able to recite the number naming sequence (i.e. count orally) to 20 and beyond; recognise, read, and write number words and numerals to 10; and count and model small collections (less than 20), will guess when asked "*how many*" in a particular collection or to identify which of two single digit numbers presented orally or in written form is the larger/smaller, and/or experience difficulty when counting larger collections (40 or more) accurately.

This could be due to/associated with:

- a failure to understand that counting is a strategy to determine "*how many*" and/or that the last number counted says how many;
- a mismatch between the oral words and the objects counted (e.g. matches objects to syllables, omits certain number names);
- a failure to organise the count to avoid counting objects already counted; and/or
- a superficial understanding of numbers 0 to 10 (i.e. limited to simple counts and recognising, reading and writing number names and numerals).

Students need a deep understanding of the numbers to 10 both in terms of what they represent and how they might be reconfigured or viewed in relation to other numbers. In particular, they need to have developed flexible mental objects for each of the numbers that go beyond the recognition of number names and numerals to include rich part-part-whole knowledge based on visual imagery. This supports trusting the count in the sense that when students read, write or hear "*seven*", they can imagine what that collection might look like and how it relates to other numbers. For example, they can see *a seven* in their mind's eye as *1 more than 6*, *1 less than 8*, *3 and 4*, or *5 and 2*. This is not about addition or subtraction. It is about deeply understanding what each number means.

A key indicator of the extent to which students have developed mental objects for the numbers 0 to 10 is the extent to which they can recognise collections of these numbers without counting, that is, they can subitise.

1. Trusting the Count

1.1 Subitising Tool

MATERIALS

- 5 Subitising Card Sets (see Trusting the Count Resources).

INSTRUCTIONS

Bold type indicates what should be said.

Say: **"I'm going to show some cards with some black dots, like this."** Show the first card of Set 1 for as long as it takes to mentally count "one and two and" (approximately 2 seconds).

Say: **"I want you to tell me how many dots there are on each card as quickly as you can".** If child did not say "two" when shown the card, ask them to say how many they saw, then proceed.

Say: **"If you can't tell me, that's okay, we'll just move on, but I want you to try as hard as you can."**

Show each card in the order listed for approximately 2 seconds starting with the second card from Set 1. Avoid making any comments.

If answered correctly place the card face down on your left (pile A).

If no response or incorrect after 2 seconds (that is, after a count of "*one-and-two-and*"), place the card face down on your right (pile B) together with any remaining cards and proceed to the next Card Set.

After the student has left, record which cards are in each pile in a table like the one below:

Student:		**Date:**
Card Set:	**Pile A**	**Pile B**
1. Single Digit (2, 4, 5, 8, 10)		
2. Ten-Frame Doubles (1, 3, 4, 6, 9)		
3. Ten-Frame To Five (3, 6, 8, 0)		
4. Ten-Frames Random (2, 4, 5, 7, 10)		
5. Two Ten-Frames (12, 14, 17, 19)		

1.2 Mental Objects Tool[1]

MATERIALS

- A small, non-transparent container with a lid (e.g. a yoghurt container or similar).
- 9 counters of the same colour (place 4 in the container and seal).
- Mental Objects Card cut and folded as indicated (see Trusting the Count Resources).

INSTRUCTIONS

Place 5 counters and the container with 4 counters in front of the student.

Say: **"There are 5 counters here and 4 more in this container."** As this is said, pick up the container and rattle it to indicate that the counters are there.

Say: **"Without opening the container, can you tell me how many counters there are altogether? How did you work that out?"**

Note student's response.

If done relatively quickly and easily, place the unfolded Mental Objects Card in front of the student, and pointing to each in turn.

Say: **"There are 7 dots here and 9 dots here."** Quickly cover the 9 dots with the flap created by cutting the card as indicated (secure if necessary).

Say: **"Without unfolding the card, can you tell me how many dots there are altogether? … How did you work that out?"**

Note student's response.

[1] Adapted from a task used by Steffe, L. P., Von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and application*. New York: Praeger.

Trusting the Count Advice

1.1 Subitising Tool

Student responses to this task indicate the extent to which they are able to recognise numbers without counting. This ability referred to as *subitising*, is an essential pre-requisite for establishing *part-part-whole number knowledge* for the numbers 1 to 10. That is, that 7 can be understood or recognised in terms of 5 and 2, 1 more than 6, 3 and 4, 1 less than 8, 3 less than 10 and so on. This task consists of 5 different card sets. Each set is ordered from smaller to larger numbers. Teachers were advised to stop and proceed to the next set as soon as the student encountered a problem (took longer than 2 seconds to respond or no response given).

Observed Response	Interpretation/Suggested Teaching Response
Little or no response (e.g. identifies first card of each set only), or clearly guessing.	*May not understand task, may not know number names, and may not recognise small collections.* • Provide opportunities to make, count, name and record small collections. • Practice one-to-one counting with appropriate materials using correct number naming sequence. • Support students to read, write and say number names and symbols.
Consistently recognise numbers up to 5 in 2 seconds or less (pile A), can occasionally recognise some numbers larger than 5 in 2 seconds (pile B).	*Able to subitise numbers to 5.* • Check that students *trust the count* for the numbers up to 5, that is, that they can work with a given number (said or read as a word or numeral) without having to make that number. • Provide opportunities to *count on from hidden*, where the collection or numeral hidden is less than/equal to 5 and the number to be counted is represented as a collection. • Develop and consolidate *part-part-whole number knowledge* for numbers to 5 (e.g. 5 is 4 and 1, 2 and 3 etc) then build on this to establish this knowledge for numbers to 10 using ten-frames and *counting on from* with particular attention to 10.

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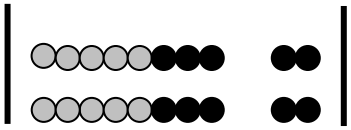
Observed Response	Interpretation/Suggested Teaching Response
Consistently recognises numbers up to 6 in 2 seconds or less (pile A) and most numbers to 10 in less than 2 seconds (pile B), may recognise some teen numbers without counting on by ones.	*Able to subitise numbers to 5, suggests part-part-whole knowledge for numbers 1 to 10 generally well established.* • Consolidate *part-part-whole number ideas* to 10 by providing plenty of practice in recognising numbers displayed quickly, use a variety of flash card displays, e.g. various ten-frame representations and cards with separate groups such as a 5 and 3, or a 2, 4 and 3, or a 5, 2, 4 and 3 (e.g. see the Subitisation Cards in Additional Resources). • Build a *sense of numbers beyond ten* by using two ten-frames or recognised representations of a group of 10 (e.g. last card in Set 1), encourage students to say, "*10 and 4 more, 14*"
Able to recognise all numbers to 10 in less than 2 seconds and most of remaining cards without counting by ones.	*Indicates a well developed capacity to subitise, a sound knowledge of part-part-whole and sense of numbers beyond 10, needs to work on mental strategies for addition to deal with larger numbers* • Use this knowledge to scaffold the *count-on-from-larger mental strategy* for single digit combinations involving 1, 2 or 3, e.g. 2 and 7 presented orally, students count on from 7 saying, 7 … 8, 9 without relying on physical models. • Once this is established, use a 2-row (2 coloured) bead frame to build a knowledge of *doubles facts to 20*, e.g.  In this case, encourage the thinking: 8 and 8 is 10 and 6 more (refer to visual image), that is, 16

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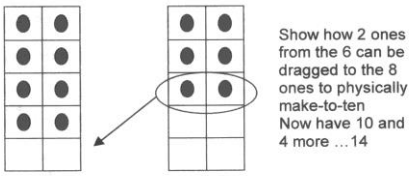
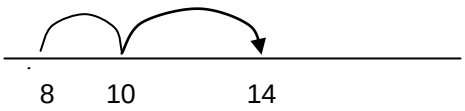
Observed Response	Interpretation/Suggested Teaching Response
Able to recognise all numbers to 10 in less than 2 seconds and most of remaining cards without counting by ones.	• Work initially with ten-frames and concrete materials to demonstrate the use of *part-part-whole knowledge* for adding numbers larger than 5, e.g. for 6 and 8, start with larger and physically make-to-ten, e.g.  • Extend to working with *Open Number Lines* (lines with no graduations/markings, this is important as the whole purpose of this is to discourage counting by ones)  Eventually, students should be able to do this mentally. Known as the make-to-ten strategy, this is quite difficult as it involves the simultaneous use of *part-part-whole knowledge*, in this case, that 8 is 2 less than 10 and 6 is a 2 and a 4

1.2 Mental Objects Tool

Student responses to this task indicate the sophistication of their counting strategies and capacity to deal with an unseen number. This task was originally employed by Steffe[2] and his colleagues at Georgia in the early 1980s to identify the steps involved in learning to count fluently and confidently but it also provides a valuable insight into the extent to which students have developed mental objects for numbers, hence the extension to larger numbers here. While the original research identified 5 different steps in the counting process, these are generally collapsed to three for the purposes of informing teaching,

(i) *perceptual counters*, where child can only count what they see and cannot deal with the hidden collection (this suggests little/no access to a range of mental objects for the hidden number);

(ii) *figural counters*, where the child can provide some sort of visual and/or auditory cue to assist with the count of the hidden collection, e.g. fingers or taps to help keep track of the count from 5 to 9 (this suggests access to some mental objects); and

(iii) *abstract or conceptual counters,* where the child immediately says "*9*" on the basis that they "*just know 9*", or number fact knowledge, e.g. "*it has to be 9, because 5 and 5 is 10*" (this suggests access to a range of mental objects for the hidden number).

Observed Response	Interpretation/Suggested Teaching Response
Little or no response, may count what they see to 5, but nothing further (*perceptual counters*).	*May not understand the task or recognise "five" without counting to confirm.* • Practice counting collections and oral counting to establish the number naming sequence. • Check and consolidate the link between collections, number words and numerals (*make, name and record* numbers to 10). • Practice *counting on from* 1, 2, or 3 using a conventional 6 –sided dot dice and another dice with 1-3 in dots and 1-3 as numerals. Toss dice, ask students to read numbers, cover 1, 2 or 3, then count on the dots on the other dice.

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[2] Steffe, L. P., Von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and application*. New York: Praeger.

1.2 Mental Objects Tool

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Observed Response	Interpretation/Suggested Teaching Response
Counts the 5 that can be seen and makes some attempt to count the hidden collection by *counting on* or *counting all* but unable to complete or incorrect.	*May not trust the count for 4, possibly still relying on perception.* • Use Subitising Cards (see Additional Resources) to encourage students to recognise small numbers without counting (*subitising*) and build *part-part-whole ideas* for numbers 1-5 (e.g. 4 is 1 and 3, 2 and 2, 1 less than 5 etc). This helps establish a *trust of the count* by developing mental objects for these numbers which support children's' efforts to count on. • Practice *counting on from given*, e.g. use a set of numeral cards and a 6- or 10-sided dice, say the number and count on dots displayed on dice. • Model counting on 2, 3 or 4 by starting from given and clapping as you count, e.g. 5 …6 (clap), 7 (clap), 8 (clap), 9 (clap). Repeat with different starting numbers and fingers or taps instead of clapping. Taps can mirror familiar pattern, e.g. if counting on 5, taps could be spatially located to represent 5 pattern on a dice.
Correctly counts on to 9 using fingers, taps or other ways of keeping track of the count (*figural counters*). Unable to deal with 7 counters task.	*Indicates access to mental object for numbers less than 5 (trusts the count for these numbers). Needs to consolidate part-part-whole ideas for numbers 1-10 and number fact knowledge.* • Use ten-frames and Subitising Cards to promote *subitising* and the development of *part-whole ideas* for the numbers 5-10 (that is, that 7 is 1 more than 6, a 5 and 2, or a 3 and 4. • Make this knowledge explicit by asking students to say what they know about a given number, e.g. "*6 is double 3*", "*it's 2 more than 4, 1 less than 7, 4 less than 10*" and so on. Record on posters and display, review regularly.

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1.2 Mental Objects Tool

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Observed Response	Interpretation/Suggested Teaching Response
Immediately correct on the basis that "*I just know" or* the use of number fact knowledge, e.g. *"I thought of 5 and 5 and 1 less made 9"* (*abstract or conceptual counters*). May attempt task with 7 counters but unable to complete or incorrect or counts on all by ones.	*Indicates sound knowledge of numbers 1-10 in terms of part-part-whole and trusting the count, needs to work on mental strategies to deal with larger numbers.* • Begin to develop mental strategies for addition (see Tool 1.1 Advice) commencing with *count on from larger* (e.g. 2 and 7, think: 7 … 8, 9). • Once this is established proceed to the *doubles and near doubles* mental strategy (e.g. 6 and 7, think: double 6 is 12 and 1 more, 13) – see Tool 1.1 Advice). • Use Ten-frames and Open number Lines (as indicated in Subitising Probe Task Advice) to scaffold the *make-to-ten* mental strategy (e.g. for 6 and 8, think: 8 … 2 more to 10 and 4 more … 14).
Answers both tasks correctly on the basis of number fact knowledge or the use of an appropriate strategy such as *make-to ten.*	*Indicates sound number fact knowledge and/or access to efficient mental strategies. Ready to proceed to further work in addition and subtraction.* • Consolidate mental strategies through practice and making strategies explicit. • Extend strategies to solve basic subtraction problems mentally, e.g. for 7 take-away 3, use part-part-whole knowledge or count back 3, for 12 take-away 5 use make-back-to-ten and part-part-whole knowledge for 5 and 10, and for 16 take-away 9 use place-value knowledge and/or halving.